ACSC/STAT 4720, Life Contingencies II

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Homework Sheet 2

Model Solutions

Basic Questions

1. *The following is a standard multiple decrement table giving probabilities of death (decrement 1) and surrender (decrement 2) for a life insurance policy:*

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
51	10000.00	19.48	3.58
52	9976.94	22.19	3.48
53	9951.27	25.24	3.38
54	9922.64	28.64	3.31
55	9890.70	32.44	3.25

A life who is in poor health has the following lifetable.

x	l_x	d_x
51	10000.00	443.73
52	9556.27	509.55
53	9046.72	579.76
54	8466.95	652.25
55	7814.70	723.74

Use this lifetable and the standard multiple decrement table to produce a multiple decrement table for this life, assuming that this life has standard surrender probabilities, using:

(a) UDD in the multiple decrement table.

Recall that for UDD in the multiple decrement table, we have ${}_tp^{01} = tp^{01}$ and ${}_tp^{02} = tp^{02}$. This gives us $\mu^{01}_{x+t} = \frac{p^{01}}{1-t(p^{01}+p^{02})}$, so the individual decrement probability is

$$p^1 = e^{-\int_0^1 \mu^1_{x+t}\,dt} = e^{-\int_0^1 \frac{p^{01}}{1-t(p^{01}+p^{02})}\,dt}$$

And we have

$$\begin{aligned}\int_0^1 \frac{p^{01}}{1-t(p^{01}+p^{02})}\,dt &= \frac{p^{01}}{p^{01}+p^{02}}\int_0^1 \frac{1}{\frac{1}{p^{01}+p^{02}}-t}\,dt\\ &= \frac{p^{01}}{p^{01}+p^{02}}\int_{\frac{1}{p^{01}+p^{02}}-1}^{\frac{1}{p^{01}+p^{02}}} \frac{1}{u}\,du\\ &= \frac{p^{01}}{p^{01}+p^{02}}\left[\log(u)\right]_{\frac{1}{p^{01}+p^{02}}-1}^{\frac{1}{p^{01}+p^{02}}}\\ &= \frac{p^{01}}{p^{01}+p^{02}}\log\left(\frac{\frac{1}{p^{01}+p^{02}}}{\frac{1}{p^{01}+p^{02}}-1}\right)\\ &= \frac{p^{01}}{p^{01}+p^{02}}\log\left(\frac{1}{1-p^{01}-p^{02}}\right)\end{aligned}$$

This gives us

$$p^1 = e^{-\frac{p^{01}}{p^{01}+p^{02}}\log\left(\frac{1}{1-p^{01}-p^{02}}\right)} = \left(\frac{1}{1-p^{01}-p^{02}}\right)^{-\frac{p^{01}}{p^{01}+p^{02}}} = \left(1-p^{01}-p^{02}\right)^{\frac{p^{01}}{p^{01}+p^{02}}}$$

Similarly

$$p^2 = \left(1-p^{01}-p^{02}\right)^{\frac{p^{02}}{p^{01}+p^{02}}}$$

Now from the multiple decrement table, the individual decrement probabilites for decrement 2 are given by:

age	p^1
51	$\left(\frac{9976.94}{10000.00}\right)^{\frac{3.58}{19.48+3.58}} = 0.999641650813$
52	$\left(\frac{9951.27}{9976.94}\right)^{\frac{3.48}{22.19+3.48}} = 0.999650807143$
53	$\left(\frac{9922.64}{9951.27}\right)^{\frac{3.38}{25.24+3.38}} = 0.999659794359$
54	$\left(\frac{9890.70}{9922.64}\right)^{\frac{3.31}{28.64+3.31}} = 0.999666041736$
55	$\left(\frac{9855.01}{9890.70}\right)^{\frac{3.25}{32.44+3.25}} = 0.999670868389$

Conversely, given the single decrement probabilities p^1 and p^2, we need to find p^{01} and p^{02} which satisfy

$$\begin{aligned}\left(1-p^{01}-p^{02}\right)^{\frac{p^{01}}{p^{01}+p^{02}}} &= p^1\\ \left(1-p^{01}-p^{02}\right)^{\frac{p^{02}}{p^{01}+p^{02}}} &= p^2\\ \left(1-p^{01}-p^{02}\right) &= p^1p^2\\ \frac{p^{01}}{p^{01}+p^{02}} &= \frac{\log(p^1)}{\log(p^1p^2)}\\ p^{01} &= (1-p^1p^2)\frac{\log(p^1)}{\log(p^1p^2)}\end{aligned}$$

This gives us the new values for p^{01} and p^{02}

age	$\frac{\log(p^2)}{\log(p^1p^2)}$	p^{01}	p^{02}
46	$\frac{\log(0.999641650813)}{\log(0.999641650813\times\frac{9556.27}{10000.00})} = 0.00783485398650$	0.0443651091518	0.000350339007267
47	$\frac{\log(0.999650807143)}{\log(0.999650807143\times\frac{9046.72}{9556.27})} = 0.00633343287128$	0.0533117881402	0.000339798723841
48	$\frac{\log(0.999659794359)}{\log(0.999659794359\times\frac{8466.95}{9046.72})} = 0.00511119732594$	0.0640754287142	0.000329184687799
49	$\frac{\log(0.999666041736)}{\log(0.999666041736\times\frac{7814.70}{8466.95})} = 0.00414935675683$	0.0770221315561	0.000320923929875
50	$\frac{\log(0.999670868389)}{\log(0.999670868389\times\frac{7090.96}{7814.70})} = 0.00337574416926$	0.0925976453981	0.000313644745962

The new multiple decrement table is therefore

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
46	10000.00	443.65	3.50
47	9552.85	509.28	3.25
48	9040.32	579.26	2.98
49	8458.08	651.46	2.71
50	7803.91	722.62	2.45

(b) UDD in the independent decrements.

Using UDD in the independent decrements, we get ${}_tp^1 = 1 - tq^1$, so $\mu^1_{x+t} = \frac{q^1}{1-tq^1}$. This gives

$${}_tp^{00} = {}_t p^1_t p^2 = (1-tq^1)(1-tq^2)$$

and thus

$$\begin{aligned} p^{01} &= \int_0^1 (1-tq^1)(1-tq^2)\frac{q^1}{(1-tq^1)}\,dt \\ &= q^1\int_0^1 (1-tq^2)\,dt \\ &= q^1\left(1-\frac{q^2}{2}\right) \end{aligned}$$

Given a multiple decrement table, we obtain the single decrement probabilities by solving

$$q^1\left(1-\frac{q^2}{2}\right)=p^{01}$$
$$q^2\left(1-\frac{q^1}{2}\right)=p^{02}$$
$$q^1-q^2=p^{01}-p^{02}$$
$$p^1p^2=p^{00}$$
$$p^1(p^1+p^{01}-p^{02})=p^{00}$$
$$p^1=\frac{p^{02}-p^{01}+\sqrt{(p^{01}-p^{02})^2+4p^{00}}}{2}$$
$$p^2=\frac{p^{01}-p^{02}+\sqrt{(p^{01}-p^{02})^2+4p^{00}}}{2}$$

Now from the multiple decrement table, the individual decrement probabilites for decrement 2 are given by:

age	q^2
51	$1-\frac{1}{2}\left(\frac{19.48-3.58}{10000.00}+\sqrt{\left(\frac{19.48-3.58}{10000.00}\right)^2+4\frac{9976.94}{10000.00}}\right)=0.000358349095$
- 52	$1-\frac{1}{2}\left(\frac{22.19-3.48}{9976.94}+\sqrt{\left(\frac{22.19-3.48}{9976.94}\right)^2+4\frac{9951.27}{9976.94}}\right)=0.000349192735$
53	$1-\frac{1}{2}\left(\frac{25.24-3.38}{9951.27}+\sqrt{\left(\frac{25.24-3.38}{9951.27}\right)^2+4\frac{9922.64}{9951.27}}\right)=0.000340589675$
- 54	$1-\frac{1}{2}\left(\frac{28.64-3.31}{9922.64}+\sqrt{\left(\frac{28.64-3.31}{9922.64}\right)^2+4\frac{9890.70}{9922.64}}\right)=0.000333558055$
55	$1-\frac{1}{2}\left(\frac{32.44-3.25}{9890.70}+\sqrt{\left(\frac{32.44-3.25}{9890.70}\right)^2+4\frac{9855.01}{9890.70}}\right)=0.000329131345$

Conversely, we use the formulae

$$p^{01}=q^1\left(1-\frac{q^2}{2}\right)$$
$$p^{02}=q^2\left(1-\frac{q^1}{2}\right)$$

To obtain the multiple decrement table:

age	p^{01}	p^{02}
51	$\frac{443.73}{10000.00}\left(1-\frac{0.000358349095}{2}\right)=0.0443650494878$	$0.000358349095\left(1-\frac{443.73}{2\times10000.00}\right)=0.000350398582804$
52	$\frac{509.55}{9556.27}\left(1-\frac{0.000349192735}{2}\right)=0.053311703669$	$0.000349192735\left(1-\frac{509.55}{2\times9556.27}\right)=0.000339883079762$
53	$\frac{579.76}{9046.72}\left(1-\frac{0.000340589675}{2}\right)=0.0640741915153$	$0.000340589675\left(1-\frac{579.76}{2\times9046.72}\right)=0.000329676312479$
54	$\frac{652.25}{8466.95}\left(1-\frac{0.000333558055}{2}\right)=0.0770219758449$	$0.000333558055\left(1-\frac{652.25}{2\times8466.95}\right)=0.00032071026203$
55	$\frac{723.74}{7814.70}\left(1-\frac{0.000329131345}{2}\right)=0.0925973994191$	$0.000329131345\left(1-\frac{723.74}{2\times7814.70}\right)1=0.00031389048357$

The new multiple decrement table is therefore

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
51	10000.00	443.65	3.50
52	9552.85	509.28	3.25
53	9040.32	579.25	2.98
54	8458.09	651.46	2.71
55	7803.92	722.62	2.45

2. *The mortalities for a husband and wife (whose lives are assumed to be independent) aged 35 and 62 respectively, are given in the following tables:*

x	l_x	d_x
35	*10000.00*	*25.33*
36	*9974.67*	*26.90*
37	*9947.77*	*28.60*
38	*9919.17*	*30.42*
39	*9888.75*	*32.38*
40	*9856.37*	*34.49*
41	*9821.88*	*36.76*
42	*9785.11*	*39.20*
43	*9745.91*	*41.82*
44	*9704.08*	*44.64*
45	*9659.45*	*47.65*

x	l_x	d_x
62	*10000.00*	*110.82*
63	*9889.18*	*117.39*
64	*9771.79*	*124.26*
65	*9647.53*	*131.42*
66	*9516.11*	*138.87*
67	*9377.24*	*146.61*
68	*9230.63*	*154.62*
69	*9076.00*	*162.89*
70	*8913.11*	*171.40*
71	*8741.71*	*180.13*
72	*8561.58*	*189.04*

The interest rate is $i = 0.04$.

(a) They want to purchase an 8-year joint life insurance policy with a death benefit of \$1,200,000. Annual premiums are payable while both are alive. Calculate the net premium for this policy using the equivalence principle.

We calculate

$$\ddot{a}_{38,62:\overline{8}|} = \frac{10000.00 \times 10000.00 + 9974.67 \times 9889.18(1.04)^{-1} + 9947.77 \times 9771.79(1.04)^{-2} + 9919.17 \times 9647.53(1.04)^{-3} + 9888.75 \times 9516.1}{100000000}$$

This gives $A_{38,62:\overline{8}|} = 1 - \frac{0.04}{1.04} \times 6.65340033009 = 0.744099987304$, so $A^{1}_{38,62:\overline{8}|} = 0.744099987304 - (1.04)^{-8} \times 0.974591 \times 0.891311 = 0.109375945763$. The premium is therefore $\frac{1200000 \times 0.109375945763}{6.65340033009} = \$19,726.93$.

(b) They want to purchase a 9-year last survivor insurance with a benefit of \$12,000,000. Premiums are payable while either life is alive. Calculate the net premium for this policy using the equivalence principle.

We calculate $\ddot{a}_{\overline{38,62}:\overline{9}|} = \frac{10000.00 \times 10000.00 + (10000 \times 10000 - (10000 - 9974.67) \times (10000 - 9889.18))(1.04)^{-1} + (10000 \times 10000 - (}{}$
7.72654919139

This gives $A^{1}_{38,62:\overline{9}|} = 1 - \frac{0.04}{1.04} \times 7.72654919139 = 0.702825031101$, so $A^{1}_{38,62:\overline{9}|} = 0.702825031101 - (1.04)^{-9} \times (1 - 0.029592 \times 0.125829) = 0.002854399552$. The premium is therefore $\frac{12000000 \times 0.002854399552}{7.72654919139} = \$4,433.13$.

3. *A husband is 76; the wife is 41. Their lifetables while both are alive, and the lifetable for the wife if the husband is dead, are given below:*

x	l_x	d_x
76	*10000.00*	*1473.82*
77	*8526.18*	*1409.36*
78	*7116.83*	*1319.45*
79	*5797.37*	*1205.58*
80	*4591.79*	*1071.07*
81	*3520.72*	*921.20*

x	l_x	d_x
41	*10000.00*	*14.70*
42	*9985.30*	*15.94*
43	*9969.37*	*17.30*
44	*9952.07*	*18.80*
45	*9933.27*	*20.44*
46	*9912.83*	*22.25*

x	l_x	d_x
41	*10000.00*	*161.87*
42	*9838.13*	*186.25*
43	*9651.88*	*213.77*
44	*9438.12*	*244.61*
45	*9193.51*	*278.89*
46	*8914.62*	*316.59*

Calculate the probability that the wife is alive in 5 years time. Use the UDD assumption for handling changes to the wife's mortality in the year of the husband's death.

Recall that under the UDD assumption, if the husband dies at time t and the probability of the wife's dying while the husband is alive is q_a, and the wife's probability of dying after the husband is dead is q_d, then the overall probability of the wife surviving is $(1 - tq_a)\frac{1-q_d}{1-tq_d}$. Conditional on the husband dying during the year, the probability of the wife surviving is

$$\begin{aligned}
\int_0^1 (1 - tq_a)\frac{1 - q_d}{1 - tq_d}\,dt &= (1 - q_d)\int_0^1 \frac{q_a}{q_d} + \frac{1 - \frac{q_a}{q_d}}{1 - tq_d}\,dt \\
&= (1 - q_d)\left(\frac{q_a}{q_d} + \frac{q_d - q_a}{(q_d)^2}\left[-\log(1 - tq_d)\right]_0^1\right) \\
&= (1 - q_d)\left(\frac{q_a}{q_d} + \frac{q_a - q_d}{(q_d)^2}\log\left(1 - q_d\right)\right)
\end{aligned}$$

This gives the following probabilities:

Year	q_a	q_d	P(wife survives given husband dies)
1	0.00147	0.016187	$(1-0.016187)\left(\frac{0.00147}{0.016187}+\frac{0.00147-0.016187}{0.016187^2}\log\left(1-0.016187\right)\right)=0.991131471483$
2	0.00159634662955	0.0189314432722	$(1-0.018931)\left(\frac{0.001596}{0.018931}+\frac{0.0015963-0.018931}{0.018931^2}\log\left(1-0.018931\right)\right)=0.989680884938$
3	0.00173531527067	0.0221480167594	$(1-0.022148)\left(\frac{0.001735}{0.022148}+\frac{0.001735-0.022148}{0.022148^2}\log\left(1-0.022148\right)\right)=0.987982138138$
4	0.00188905423696	0.025917237755	$(1-0.025917)\left(\frac{0.001889}{0.025917}+\frac{0.001889-0.025917}{0.025917^2}\log\left(1-0.025917\right)\right)=0.985991697046$
5	0.00205773124057	0.0303355301729	$(1-0.030336)\left(\frac{0.002058}{0.030336}+\frac{0.002058-0.030336}{0.030336^2}\log\left(1-0.030336\right)\right)=0.983658190129$

We now calculate the wife's overall survival probability:

Year	P(Husband dies)	P(W survives to start)	P(W survives)	P(W survives from end)	P(W survives)
1	0.147382	1.000000	0.991131471483	$\frac{8914.62}{9838.13}=0.906129518516$	0.132362813718
2	0.140936	0.998530	0.989680884938	$\frac{8914.62}{9651.88}=0.923614881246$	0.128637965447
3	0.131945	0.996937	0.987982138138	$\frac{8914.62}{9438.12}=0.94453344522$	0.122751578509
4	0.120558	0.995207	0.985991697046	$\frac{8914.62}{9193.51}=0.969664469827$	0.114710770555
5	0.107107	0.993327	0.983658190129	$\frac{8914.62}{8914.62}=1$	0.104653632659
>5	0.352072	0.991283	1	1	0.349002988376

The total probability is therefore

$$0.132362813718+0.128637965447+0.122751578509+0.114710770555+0.104653632659+0.349002988376=0.952119749264$$

Standard Questions

4. *The following is a multiple decrement table giving probabilities of surrender (decrement 1) and death (decrement 2) for a life insurance policy:*

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
44	*10000.00*	*21.36*	*6.74*
45	*9971.90*	*19.85*	*11.25*
46	*9940.80*	*18.47*	*15.95*
47	*9906.39*	*17.21*	*20.89*
48	*9868.29*	*16.08*	*26.10*
49	*9826.11*	*15.04*	*31.65*
50	*9779.42*	*14.10*	*37.59*
51	*9727.73*	*13.24*	*43.97*
52	*9670.52*	*12.46*	*50.83*
53	*9607.23*	*11.75*	*58.25*
54	*9537.23*	*11.10*	*66.28*

A life insurance policy pays a benefit of $640,000 at the end of the year of death. Premiums are payable at the beginning of each year. Calculate the premium for a 10-year policy sold to a life aged 44 if the interest rate is $i=0.08$*.*

We calculate

$$
\begin{aligned}
A^{02}_{44:\overline{10|}} &= 0.000674(1.08)^{-1} + 0.001125(1.08)^{-2} + 0.001595(1.08)^{-3} + 0.002089(1.08)^{-4} + 0.002610(1.08)^{-5} + 0.003165(1.08)^{-6} \\
&\quad + 0.003759(1.08)^{-7} + 0.004397(1.08)^{-8} + 0.005083(1.08)^{-9} + 0.005825(1.08)^{-10} \\
&= 0.0179707992962
\end{aligned}
$$

and

$$
\begin{aligned}
\ddot{a}^{00}_{51:\overline{10|}} &= 1 + 0.997190(1.08)^{-1} + 0.994080(1.08)^{-2} + 0.990639(1.08)^{-3} + 0.986829(1.08)^{-4} + 0.982611(1.08)^{-5} + 0.977942(1.08)^{-6} \\
&\quad + 0.972773(1.08)^{-7} + 0.967052(1.08)^{-8} + 0.960723(1.08)^{-9} \\
&= 7.14302777767
\end{aligned}
$$

so the premium is $\frac{640000\times 0.00971950241384}{7.39147970569} = \$368.19$.

5. *A couple want to receive the following:*

   - *While both are alive, they would like to receive a pension of \$140,000 per year.*
   - *If the husband is alive and the wife is not, they would like to receive a pension of \$90,000 per year.*
   - *If the wife is alive and the husband is not, they would like to receive a pension of \$70,000 per year.*
   - *When the husband dies: if the wife is still alive, they would like a death benefit of \$700,000; otherwise, they would like a death benefit of \$300,000.*
   - *When the wife dies: if the husband is still alive, they would like a death benefit of \$400,000; otherwise, they would like a death benefit of \$200,000.*

   Construct a combination of insurance and annuity policies that achieve this combination of benefits.

   There are a number of possible solutions. Here is one example:

   - A life annuity for the husband of \$20,000 per year.
   - A joint life annuity of \$50,000 per year.
   - A last survivor annuity of \$70,000 per year.
   - A joint life insurance of \$200,000.
   - A contingent insurance policy of \$200,000 payable if the husband dies while the wife is still alive.
   - A life insurance policy of \$200,000 on the wife.

- A life insurance policy of $300,000 on the husband.

This meets the needs of the couple as shown in the following table:

	Both alive	H alive W dead	W alive H dead	W dies H alive	W dies H dead	H dies W alive	H dies W dead
life annuity (H)	$20,000	$20,000	$0	$0	$0	$0	$0
joint life annuity	$50,000	$0	$0	$0	$0	$0	$0
last survivor annuity	$70,000	$70,000	$70,000	$0	$0	$0	$0
life insurance (W)	$0	$0	$0	$200,000	$200,000	$0	$0
life insurance (H)	$0	$0	$0	$0	$0	$300,000	$300,000
joint life insurance	$0	$0	$0	$200,000	$0	$200,000	$0
contingent insurance	$0	$0	$0	$0	$0	$200,000	$0
Total	$140,000	$90,000	$70,000	$400,000	$200,000	$700,000	$300,000

6. *A husband aged 61 and wife aged 54 have the following transition intensities:*

$$\begin{aligned}\mu_{xy}^{01} &= 0.005y - 0.09\\ \mu_{xy}^{02} &= 0.003x - 0.027\\ \mu_{xy}^{03} &= 0.08\\ \mu_{x}^{13} &= 0.006x - 0.022\\ \mu_{y}^{23} &= 0.012y - 0.024\end{aligned}$$

They want to purchase a last survivor insurance, which will pay a benefit of $1,700,000 when the second life dies. Premiums are payable continuously while either life is alive. Force of interest is $\delta = 0.04$.

(a) Calculate the annual rate of continuous premium.

We calculate $\overline{a}_{\overline{x,y}} = \overline{a}_{x,y}^{00} + \overline{a}_{x,y}^{01} + \overline{a}_{x,y}^{02}$.

We have

$${}_tp_{61,54}^{00} = e^{-\int_0^t 0.008s+0.005\times 54+0.003\times 61+0.08-0.09-0.027\,ds} = e^{-\int_0^t 0.008s+0.416\,ds} = e^{-0.004t^2-0.416t}$$

$${}_{t-s}p_{61+s}^{11} = e^{-\int_s^t 0.006(61+u)-0.022\,du} = e^{-0.003(t^2-s^2)-0.344(t-s)}$$

$${}_{t-s}p_{54+s}^{22} = e^{-\int_s^t 0.012(54+u)-0.024\,du} = e^{-0.006(t^2-s^2)-0.624(t-s)}$$

$$
\begin{aligned}
\overline{a}^{00}_{61,54} &= \int_0^\infty e^{-0.04t} e^{-0.004t^2-0.416t} \\
&= \int_0^\infty e^{-0.004t^2-0.456t}\,dt \\
&= \int_0^\infty e^{-0.004(t^2+2\times 57t)}\,dt \\
&= \sqrt{\frac{\pi}{0.004}} e^{12.996}\left(1-\Phi\left(57\sqrt{0.008}\right)\right) \\
&= 2.116855778
\end{aligned}
$$

Next we calculate

$$
\begin{aligned}
{}_tp^{01}_{61,54} &= \int_0^t e^{-0.004s^2-0.416s}(0.005(54+s)-0.09)e^{-0.003(t^2-s^2)-0.344(t-s)}\,ds \\
&= e^{-0.003t^2-0.344t}\int_0^t (0.18+0.005s)e^{-0.001s^2-0.072s}\,ds \\
&= e^{-0.003t^2-0.344t}\int_0^t (0.18+0.005s)e^{-0.001(s+36)^2+1.296}\,ds \\
&= e^{-0.003t^2-0.344t+1.296}\int_0^t 0.005(s+36)e^{-0.001(s+36)^2}\,ds \\
&= e^{-0.003t^2-0.344t+1.296}\left[-2.5e^{-0.001(s+36)^2}\right]_0^t \\
&= 2.5e^{-0.003t^2-0.344t+1.296}\left(1-e^{-0.001(t+36)^2}\right) \\
&= 2.5\left(e^{-0.003t^2-0.344t+1.296}-e^{-0.004t^2-0.416t}\right)
\end{aligned}
$$

Integrating this gives

$$
\begin{aligned}
\overline{a}^{01} &= 2.5\int_0^\infty e^{-0.04t}\left(e^{-0.003t^2-0.344t+1.296}-e^{-0.004t^2-0.416t}\right)dt \\
&= 2.5\int_0^\infty \left(e^{-0.003t^2-0.384t+1.296}-e^{-0.004t^2-0.456t}\right)dt \\
&= 2.5\left(\int_0^\infty e^{-0.003t^2-0.384t+1.296}\,dt-\int_0^\infty e^{-0.004t^2-0.456t}\,dt\right) \\
&= 2.5\left(\int_0^\infty e^{-0.003(t+64)^2+12.288}\,dt-\int_0^\infty e^{-0.004(t+57)^2+12.996}\,dt\right) \\
&= 2.5\left(e^{12.288}\sqrt{\frac{\pi}{0.003}}\left(1-\Phi\left(64\sqrt{0.006}\right)\right)-e^{12.996}\sqrt{\frac{\pi}{0.004}}\left(1-\Phi\left(57\sqrt{0.008}\right)\right)\right) \\
&= 0.9805182694
\end{aligned}
$$

Similarly

$$\begin{aligned}
{}_tp^{02}_{61,54} &= \int_0^t e^{-0.004s^2-0.416s}(0.003(61+s)-0.027)e^{-0.006(t^2-s^2)-0.624(t-s)}\,ds \\
&= \int_0^t e^{-0.004s^2-0.416s}0.003(s+52)e^{-0.006(t^2-s^2)-0.624(t-s)}\,ds \\
&= e^{-0.006t^2-0.624t}\int_0^t 0.003(s+52)e^{0.002s^2+0.208s}\,ds \\
&= e^{-0.006t^2-0.624t}\int_0^t 0.003(s+52)e^{0.002(s+52)^2-5.408}\,ds \\
&= e^{-0.006t^2-0.624t-5.408}\left[0.75e^{0.002(s+52)^2}\right]_0^t \\
&= 0.75e^{-0.006t^2-0.624t-5.408}\left(e^{0.002(t+52)^2}-1\right) \\
&= 0.75\left(e^{-0.004t^2-0.416t}-e^{-0.006t^2-0.624t-5.408}\right)
\end{aligned}$$

Integrating this gives

$$\begin{aligned}
\overline{a}^{02} &= 0.75\int_0^\infty e^{-0.04t}\left(e^{-0.004t^2-0.416t}-e^{-0.006t^2-0.624t-5.408}\right)dt \\
&= 0.75\int_0^\infty \left(e^{-0.004t^2-0.456t}-e^{-0.006t^2-0.664t-5.408}\right)dt \\
&= 0.75\left(\int_0^\infty e^{-0.004(t+57)^2+12.996}\,dt-\int_0^\infty e^{-0.006(t+55.3333333333333)^2+12.96266666667}\,dt\right) \\
&= 0.75\left(e^{12.996}\sqrt{\frac{\pi}{0.004}}\left(1-\Phi\left(57\sqrt{0.008}\right)\right)-e^{12.96266666667}\sqrt{\frac{\pi}{0.006}}\left(1-\Phi\left(55.3333333333333\sqrt{0.012}\right)\right)\right) \\
&= 1.582708715
\end{aligned}$$

This gives $\overline{a}_{\overline{61,54}} = 2.116855778+0.9805182694+1.582708715 = 4.6800827624$
We have

$$\overline{A}_{\overline{61,54}} = 1-\delta\overline{a}_{\overline{68,75}} = 1-0.04\times 4.6800827624 = 0.812796689504$$

The annual rate of premium is therefore $\frac{1700000\times 0.812796689504}{4.6800827624} = \$295,241.439588$

(b) Calculate the policy value after 3 years if the husband is dead and the wife is alive.

We compute:

$${}_tp^{22}_{57} = e^{-\int_0^t 0.012(s+57)-0.024\,ds} = e^{-0.006t^2-0.66t}$$

We therefore get

$$\begin{aligned}
\overline{a}_{57}^{22} &= \int_0^\infty e^{-0.04t} e^{-0.006t^2-0.66t}\,dt \\
&= \int_0^\infty e^{-0.006t^2-0.7t}\,dt \\
&= e^{20.4166666666} \int_0^\infty e^{-0.006(t+58.3333333333333)^2}\,dt \\
&= e^{20.4166666666} \sqrt{\frac{\pi}{0.006}} \left(1-\Phi\left(58.3333333333333\sqrt{0.012}\right)\right) \\
&= 1.395886139
\end{aligned}$$

so $\overline{A}_{57} = 1 - 0.04 \times 1.395886139 = 0.94416455444$.

The policy value after 3 years is therefore $1700000 \times 0.94416455444 - 295241.439588 \times 1.395886139 = \$1,192,956.31$.