

**ECVar**(**x**)

**x** is a vector of observed values. We have parameters **a**, **N**, **T**, which in our implementation is fixed at 9 and **uselogs** for the log transformation. Recall that we compute  $g(x)$  is the coefficient of  $\lambda^x$  in  $\log(a) + \sum_{i=0}^N \frac{(-1)^i}{i} \left(\frac{\lambda}{a} - 1\right)^i$  for  $x < \text{uselogs}$ , and  $g(x) = \log(x)$  for  $x > \text{uselogs}$ .

We compute the value **ECVar**(**x**) as  $\sum_{i=1}^n h(x_i)$ . We precompute the values of  $h(x)$  for  $0 \leq x \leq \text{uselogs}$ . We use a variant of Stirling numbers given by the recurrence

$$\begin{aligned} \begin{Bmatrix} i \\ 1 \end{Bmatrix} &= 1 \\ \begin{Bmatrix} 1 \\ i \end{Bmatrix} &= 0 \\ \begin{Bmatrix} i+1 \\ j \end{Bmatrix} &= j \begin{Bmatrix} i \\ j \end{Bmatrix} + i \begin{Bmatrix} i-1 \\ j-1 \end{Bmatrix} \end{aligned}$$

These count the number of ways to partition a set of size  $i$  into  $j$  partitions of size at least 2.

We calculate coefficients  $c_{i,j}$  by

$$c_{i,j} = \sum_{k=0}^i \sum_{l=0}^j \frac{1}{k(i-k)} \begin{Bmatrix} k+1 \\ l+1 \end{Bmatrix} \begin{Bmatrix} i-k \\ j-l \end{Bmatrix}$$

We compute the values  $s_i$  given by

$$\begin{aligned} s_0 &= 1 \\ s_i &= \frac{(i+1)^2 s_{i-1} + 2}{(i+1)(i+2)} \end{aligned}$$

Next we compute coefficients given by

$$a_i = \sum_{j=0}^{i \wedge \frac{N+1}{2}} s_{i+1-j} \begin{Bmatrix} i+1-j \\ j \end{Bmatrix} - c_{i+1-j,j}$$

Finally, we compute

$$h(x) = \sum_{j=1}^N \frac{a_j}{\prod_{i=1}^j (x+i)}$$