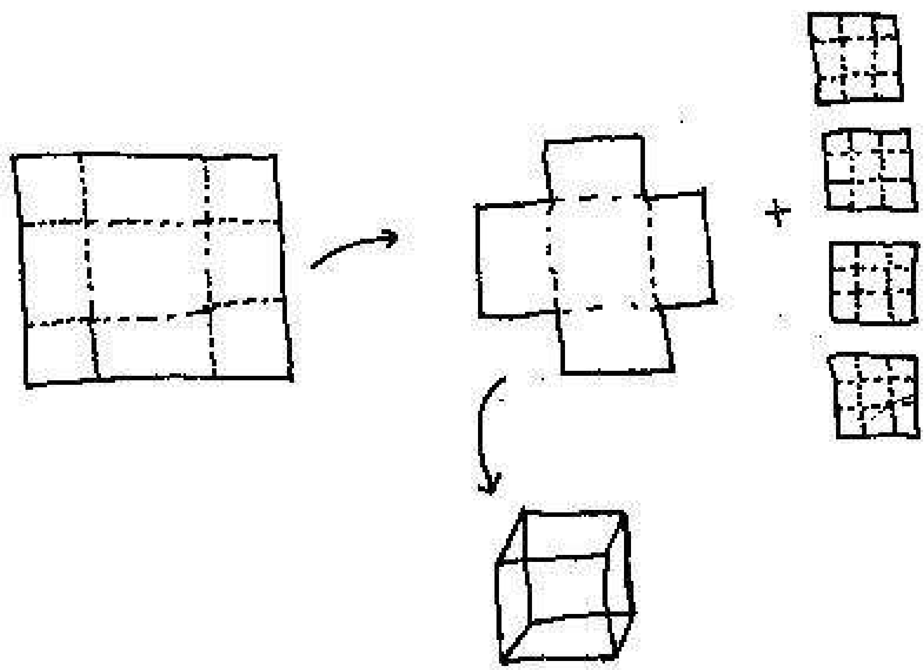
Bonus Question 5

1. Do parts 1 and 2 of "the shape of the can" problem from the textbook, p.341.
2. Take a square sheet of paper whose side is of length l. From each of the four corners, cut out a smaller square each of whose length is a fraction r_1 of l, with $0 < r_1 < 1/2$. From the resulting cross, make a box of height lr_1 and of base length $l\,(1-2r_1)$. For each of the remaining four squares, cut out four corners, each of whose length is a fraction r_2 of the length of the small square. Then make four additional boxes from the four resulting crosses.



a) How should r_1 and r_2 be chosen in order to maximize the total volume of the resulting five boxes?

b) Continue the procedure indefinitely, defining a sequence of ratios $r_1, r_2, r_3 \ldots$ and resulting in $1, 5, 21, \ldots$ boxes. Suppose that it is required that all these ratios are the same: $r = r_1 = r_2 = r_3 = \ldots$. How should you choose r in order to maximize the total volume?

c) Now suppose that you are free to choose $r_1, r_2, r_3 \ldots$ independently from one another, in such a way as to maximize the total volume. Would you get a different value for r_1 than what you found for r in part b? If so, what would it be?

Hint: you may find the following formula (geometric series) useful:

$$1 + a + a^2 + a^3 + \cdots = \frac{1}{1-a}, \quad |a| < 1.$$