MATH 1500, Homework 6

1. Find real numbers x, y such that $z = x + iy$ where (a) $z = \frac{2}{3-4i}$. (b) $z = e^{3+i\pi/4}$.

2. (a) Let $z = 1 + \sqrt{3}i$. Find r and θ such that $z = re^{i\theta}$. (b) What is $\left(1 + \sqrt{3}i\right)^{2015}$?

3. Determine all the roots of the polynomial $z^5 - i = 0$, and sketch them in a complex plane.

4. (a) Find the general solution to the ODE $y'' - y' - 2y = 0$. (b) Find the solution to this ODE subject to initial conditions $y(0) = 0$, $y'(0) = -1$.

5. (a) Find the general solution to the ODE $4y'' - 4y' + y = 0$. (b) Find the solution to this ODE that in addition satisfies initial conditions $y(0) = 1$, $y'(0) = 2$.

6. (a) Find the general solution to the ODE $y'' - 4y' + 5y = 0$. (b) Find the solution to this ODE that in addition satisfies initial conditions $y(0) = 1$, $y'(0) = 2$.

7. (a) Write down a second order linear ODE that has the following particular solutions: $y_1 = e^t$, $y_2 = e^{-2t}$. (b) Write down a second order linear ODE that has the following particular solution: $y_1 = \cos(t)e^t$. What is another independent particular solution?

8. (a) Find the general solution to the ODE $y''' - y' = 0$. (b) Find the solution to this ODE subject to initial conditions $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$.

9. A mass of $m = 0.1$ kg is attached to a spring. The spring constant is known to be $k = 50N/m$, and you wish to find the friction constant c; recall that the mass motion satisfies $mx'' + cx' + kx = 0$. To find the spring constant, you observe the spring oscillates with a frequency of 3 Hertz (i.e. three full oscillations per second). Determine c.