Sample midterm 3 questions

1. Consider the region bounded by curves $x = 2y^2$ and $x + y = 1$. Sketch this region and find its centroid. Find the volume obtained by rotating this region around the y axis and the x axis.

2. A pyramid has height h and has a square base of length l. Find its volume.

3. Find the centroid of an arc of a unit circle suspended by angle θ.

4. A cone is cut out of a unit sphere. The vertex of the cone is at the center of the sphere; the cone angle is given by θ. Find the volume of the resulting cone.

5. For the cone as in question 4, find the area of the region that lies on the surface of the sphere and is inside the given cone.

6. Find the work that must be done to pump all the water in a full hemispherical bowl of radius a meters to a height h meters above the top of the bowl.

7. Evaluate the following integrals:

$$(a)\ \int x^2 e^x dx \qquad (b)\ \int_0^1 x\sqrt{1-x}dx \qquad (c)\ \int_0^{\pi/2} \sin^3 x\ dx$$

$$(d)\ \int \arcsin x\ dx \qquad (e)\ \int_0^{\pi/3} \tan^4(x) \qquad (f)\ \int \frac{2x^4+5x^3+4x^2+x+2}{x^3+2x^2}dx$$

$$(g)\ \int \frac{x^2}{(16-x^2)^{3/2}}dx \qquad (h)\ \int \frac{1}{(4x^2-8x+8)^2}dx \qquad (i)\ \int_0^{1/2} \sqrt{x-x^2}$$

8. Determine whether the following integrals converge or diverge (you do not need to evaluate them):

$$(a)\ \int_0^\infty \frac{\sqrt{x}}{x^2+1}dx \qquad (b)\ \int_0^1 \frac{e^x}{1-x}dx \qquad (c)\ \int_0^1 \frac{\sqrt{x}}{\sin(x)}dx \qquad (d)\ \int_0^\infty \frac{e^{-x}}{(x+x^2)^{1/2}}$$

9.

10. Use Newton's method to find $\ln(0.5)$.

11. Consider the map $x_{n+1} = a \tan x_n$.

    (a) Note that $x = 0$ is a fixed point of this map. For which values of a is this fixed point stable?

    (b) Show that this map has a fixed point $x \in (\pi k + \frac{\pi}{2}, \pi k + \frac{\pi}{2} + \pi)$ for any integer k. (hint: make a graph).

    (c) Fix $a = 2$. Find a fixed point of this map that is located in the interval $\left(\frac{\pi}{2}, \frac{3}{2}\pi\right)$.

12. (a) Compute the integral $\int_0^1 e^{-t^2} dt$ using (i) Midpoint rule with $n = 2$ [answer: 0.7788]; (ii) Trapezoid rule with $n = 2$ and [answer: 0.731]; (iii) Simpson's rule with $n = 4$. [Answer: 0.747].

    (b) Estimate the error when evaluating $\int_0^1 e^{-t^2} dt$ using Midpoint rule with $n = 2$. Note: the error for midpoint rule is bounded by $\frac{M}{24}h^2(b-a)$ where $M \geq \max_{[a,b]} |f''(x)|$.

    (c) Consider $F(x) = \int_0^x e^{-t^2} dt$. Find a Taylor series for F. Estimate $F(1)$ using three terms of the Taylor series expansion of $F(x)$ around $x = 0$. Estimate the error.

    (d) You want to compute $\int_0^\infty e^{-t^2} dt$ to within 10^{-2}. Write it as $\int_0^\infty e^{-t^2} dt = \int_0^M e^{-t^2} dt + \int_M^\infty e^{-t^2} dt$. Show how to choose M so that $\int_M^\infty e^{-t^2} dt \leq \frac{1}{2}10^{-2}$. Next, estimate $\int_0^M e^{-t^2}$ using any method you like, to within $\frac{1}{2}10^{-2}$. Combine these results to compute $\int_0^\infty e^{-t^2} dt$ to within 10^{-2}.

13. A certain integral was computed using midpoint rule M_n with $n = 1, 2, 4$. It was determined that $M_1 = 0.8535$, $M_2 = 0.8981$, $M_4 = 0.9069$. Use Romberg integration to find this integral as accurately as you can [Answer: 0.90962].