MATH 2300, Homework 2

1. Consider the dynamical $x_{n+2} = x_n - 2x_{n+1}$ with $x_0 = 0$ and $x_1 = 1$.

   (a) Compute x_1 to x_5 (by hand).

   (b) Using any method you like, find an explicit formula for x_n. Verify your formula by comparing to part (a).

   (c) Using formula in part (b), estimate $\log(|x_{2300}|)$.

2. An insect population is divided into three age classes, ages 0,1 and 2. During each time period 20% of the females of age 0 and 70% of the females of age 1 survive until the end of the following breeding season. Females of age 1 have an average fecundity of 3.2 offspring per female and the average fecundity of age 2 females is 1.7. No female lives beyond three years.

   (a) Draw a transition diagram and set up a dynamical system to describe this situation.

   (b) If initially the population consists of 2000 females of age 0, 800 females of age 1, and 200 females of age 2, find the age distribution after three years.

   (c) Use Matlab to plot the population of each age class in years 0 to 20.

   (d) Use the matlab command `eig` to compute the relevant eigenvalues and answer the following questions. (d1) Is the population sustainable? In other words, does it survive as $t \to \infty$? (d2) Determine the long-time distribution the population. In other words, what fraction of the population belongs to each class after a long time?

3. A regional rent-a-car company has offices in Lincoln, Omaha and ten other small cities. Their records show that each week, 55% of the cars rented in Lincoln returned to Lincoln, 35% are returned to Omaha and the remaining cars are returned elsewhere. Of those rented in Omaha, 75% are returned to Omaha, 5% to Lincoln and 20% elsewhere. Of the cars rented in the ten other cities, 25% are returned to Omaha and 5% to Lincoln.

   (a) Draw a graph illustrating the states and the transition probabilities.

   (b) Set up a dynamical system (using transition matrices) which models this situation. Using Matlab, simulate your system. Start with 100% of the cars in Lincoln, and output the fraction of the cars in each of Lincoln, Omaha and elsewhere each week. How many weeks are required until the system is roughly at the steady state? What is that steady state? Hand in your code and output.

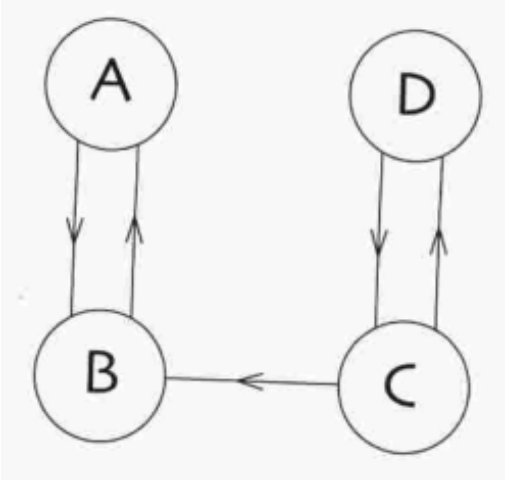
   (c) Compute the steady state analytically.

4. Two balls are distributed between two urns labelled A and B. Each turn one of the two balls is selected at random with equal probability and moved to the other urn.

   (a) Formulate this problem as a dynamical system for the three possible states of urn A. Dont forget to sketch a transition diagram.

   (b) In the long run, what fraction of the time is urn A empty?

5. Consider the following web graph:



   (a) Find the page ranking assigned by the simple PageRank algorithm (random web walker following links only).

   (b) You should observe that PageRank of C, D is zero. Explain why. To better rank pages, the transition matrix used by an improved PageRank algorithm is

$$A' = \beta A + (1 - \beta)\, Q$$

   where A is the transition matrix as in part (a), and Q is the 4x4 matrix all of whose entries are $1/4$. (this corresponds to random walker following links a fraction β of the time, and jumping at random a fraction $1 - \beta$ of the time). Determine PageRank with $\beta = 0, 0.25, 0.5, 0.75$ and 1. Using computer or otherwise, determine the value of β such that the walker spend one-third of his time visiting the pair (C, D).

6. Question 6 on page 138 (photocopy machine model)

7. Question 7 on same page (refined model of photocopy machine)