MATH 2300, Homework 3

1. Smith is in jail and has 3 dollars; he can get out on bail if he has 8 dollars. A guard agrees to make a series of bets with him. If Smith bets A dollars, he wins A dollars with probability 0.4 and loses A dollars with probability 0.6. Find the probability that he wins 8 dollars before losing all of his money if

   (a) he bets 1 dollar each time (timid strategy).

   (b) he bets, each time, as much as possible but not more than necessary to bring his fortune up to 8 dollars (bold strategy).

   (c) Which strategy gives Smith the better chance of getting out of jail?

2. With the situation in Exercise 1, consider the strategy such that for $i < 4$, Smith bets $min(i, 4-i)$, and for $i \geq 4$ he bets according to the bold strategy, where i is his current fortune. Find the probability that he gets out of jail using this strategy. How does this probability compare with that obtained for the bold strategy?

3. Assume that a student going to a certain four-year medical school has, each year, a probability q of flunking out, a probability r of having to repeat the year, and a probability p of moving on to the next year (in the fourth year, moving on means graduating).

   (a) Form a transition matrix for this process taking as states 1, 2, 3, 4, F and G where F stands for flunking out and G for graduating, and the other states represent the year of study.

   (b) For the case q = .1, r = .2, and p = .7 find the time a beginning student can expect to be in the second year. How long should this student expect to be in medical school?

   (c) Find the probability that this beginning student will graduate.

4. Consider the predator-prey model from page 36 of the book:

$$D_{n+1} = R\left(1 - \frac{D_n}{K}\right) D_n + D_n - P_n D_n$$
$$P_{n+1} = Q D_n P_n.$$

   Here, D_n is the density of prey and P_n is the population of predator at time n.

   (a) Using Matlab, plot the first 70 generations of D_n and P_n with the following parameter values:

$$K = 100, \ R = 1.5, \ Q = 0.023 \tag{1}$$

   with $D_0 = 50$, $P_0 = 0.2$. Hand in your code and the graph. You should observe oscillations of both prey and predator for these parameter values. Note: if you wish, you can modify the code we used for the plant-deer interactions in class, I posted that code on the website.

   (b) Try decreasing the parameter Q (the measure of the efficiency of utilization of prey for reproduction by predators), while keeping all other parameters as in part (a). Investigate what happens to the oscillatory solution as you do this. You will find that there exists a critical threshold Q_c which separates the oscillatory from non-oscillatory regime. Estimate the value of Q_c numerically using the matlab code you wrote in (a). Hand in the appropriate graphs for various values of Q showning your estimates.

   (c) Determine the non-zero steady state of this system, for general K, R, Q.

   (d) For general K, R, Q, determine a 2x2 matrix whose eigenvalues determine the stability of the system.

   (e) Compute the eigenvalues of this matrix for parameter values in (1) (give their numerical values). Comment on whether these eigenvalues agree with the observed behaviour of the system.

   (f) When $K = 100, R = 1.5$ and $Q = Q_c$ (the value you obtained numerically in (b)), what are the corresponding eigenvalues? How does it explain the threshold value of Q_c?

   (g) For general parameter values of K, R, determine Q_c as a function of K and R. Hint: Determinant is the product of the eigenvalues. What should be the determinant when $Q = Q_c$?