Some additional practice/sample midterm questions

- NOTE: at least two questions on the midterm will be very similar to some of these.
- Please bring a calculator with you. You are allowed one piece of chit-sheet on the midterm. Unfortunately no computers are allowed.
- Review/do all the questions on homeworks 1, 2, 3.

1. Write (on paper) a Matlab program to compute $n!$ with $n = 1 \ldots 10$, and then to plot n versus $n!$ with $n = 1 \ldots 10$. Recall that $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot\ n$.

2. Given a dynamical system
$$x_{n+1} = 5 + 0.5x_n,$$
with $x_0 = 0$, write down an explicit formula for x_n. Show that $x_n \to -10$ as $n \to \infty$.

3. Consider the iteration $x_{k+1} = f(x_k)$ where $f(x) = (a+1)x - x^2$.

   (a) Verify that this iteration has a fixed point $x = a$.

   (b) Suppose that x_0 is very close to a. For which values of a is it true that $x_k \to a$ as $k \to \infty$?

4. You take a bank loan worth \$12K. Every month, you make a payment of \$1K at the *end* of each month. The interest rate is 1% per month, compounded daily. How much do you owe after 12 months?

5. The Land of Oz is blessed by many things, but not by good weather. They never have two nice days in a row. If they have a nice day, they are just as likely to have snow as rain the next day. If they have snow or rain, they have an even chance of having the same the next day. If there is change from snow or rain, only half of the time is this a change to a nice day.

   (a) Model this situation as a Markov process. What are the states? Draw a transition diagram and find the transition matrix T.

   (b) It is sunny on Monday. What are the chances it will be sunny on Wednesday? On Thursday?

   (b) How often does it snow, rain, or sunny in the land of Oz?

6. A professor with four umbrellas resides on campus. Every morning if it is raining he takes one umbrella and walks to his office. However when all four umbrellas are at one place (home or office alike), even if there is no rain, he will take one umbrella to his office. As he moves from his office back home, uses the same approach. Also assume that probability of rain (in the morning or in the evening) remains the same (morning or evening, day after day) and is equal to p (where $0 < \text{p} < 1$).

   (a) Model this situation as a Markov process. What are the states? Draw a transition diagram and find the transition matrix T.

   (b) Find the steady state distribution of the number of umbrellas at home.

   (c) What is the expected number of umbrellas at professor's residence?

7. Two players, A and B, play the game of matching pennies: at each time n, each player has a penny and must secretly turn the penny to heads or tails. The players then reveal their choices simultaneously. If the pennies match (both heads or both tails), Player A wins the penny. If the pennies do not match (one heads and one tails), Player B wins the penny. Suppose the players have between them a total of 5 pennies. If at any time one player has all of the pennies, to keep the game going, he gives one back to the other player and the game will continue.

   (a) Model this situation as a Markov process. What are the states? Draw a transition diagram and find the transition matrix T.

(b) If Player A starts with 3 pennies and Player B with 2, what is the probability that A will lose his pennies first?

(c) If Player A starts with 3 pennies and Player B with 2, how long is the game expected to last?

8. A nonlinear system $\begin{cases} x_{n+1} = f(x_n, y_n) \\ y_{n+1} = g(x_n, y_n) \end{cases}$ has a steady state $x = 0, y = 0$. Moreover it is known that

$$\begin{bmatrix} f_x(0,0) & f_y(0,0) \\ g_x(0,0) & g_y(0,0) \end{bmatrix} = \begin{bmatrix} 0 & a \\ 1 & 0 \end{bmatrix}$$

where a is the bifurcation parameter in the model. Characterize the stability of the steady state as a function of a. In other words, for which values of a is the steady state stable and for which it is unstable? What are threshold values of a?