Course overview

Course outline:

- Compound interest, mortgage calculations
- dynamical systems:
  - steady state
  - stability
  - in 2d: linearization, eigenvalues, relation to stability
- Markov processes/transition matrices
  - eigenvalues
  - setting up transition matrices
  - computing steady states
  - Terminal states, fundamental matrix, exit probabilities
- Probability
  - binomial, normal distribution; approximating binomial by normal,
  - survey accuracy
  - Poisson distrbution
- Queueing theory
  - setting up probabilities and stationary probabilities
  - waiting times/queue length
- Least squares
  - line of best fit, R2
  - Least squares for other situations; transforming the problem.

Do all the questions on all of the homework.
Do all the questions from the sample midterm list/midterm.
Additonal questions:

1. The harmonic numbers H_n are defined as $H_n = 1 + 1/2 + \ldots + 1/n$. Write a Matlab program to compute H_n for $n = 1 \ldots 30$ and then to plot H_n as function of n.
2. Each month, I contribute \$100 to a savings account which accumulates interest at 6% per year.
   - How much money will the account have after 10 years worth of contributions?
   - After 10 years, I start using the account, withdrawing X dollars per month. I emptied the account in five years. How big is X?
3. Consider the dynamical system
$$P_{n+1} = \frac{aP_i}{1 + P_i}$$
where $a > 0$ is a constant. Find all equilibria of this system. For each equilibrium found, determine the values of a for which it is stable.

4. A dynamical system is of the form
$$x_{n+1} = (A + b)\, x_n$$
where x_n is a 2x1 vector, A is a 2x2 matrix, and b is a scalar. The matrix A has eigenvalues $\lambda_1 = 0.3 + 0.5i$ and $\lambda_2 = 0.3 - 0.5i$.
   - Suppose that $b = 0$. Is the steady state $x = 0$ stable or unstable?
   - Determine the range of b for which $x = 0$ is stable and for which it unstable.
5. **Markov chains:** See a list of 100 problems here:
   - `http://www2.math.uu.se/~takis/L/McRw//SPEX/spex.pdf`
   - Do problems 1-10 from this list.
6. In a survey of 1000 people, 550 respondents showed a preference for candidate A over candidate B. What are the chances that candidate A wins? What are the chances that A wins 60% or more of all the votes? Note: use normal approximation to binomial distribution to do this question.
7. On average, there is one car crash per day on a certain section of a certain highway. What are the chances that there will be exactly 10 crashes in one week? What are the chances of having three or more crashes in two consecutive days?
8. A restaurant has 8 tables and on average each table is occupied for about an hour. People arrive at a rate of one table every ten minutes, unless no table is available, in which case the restaurant loses business. Compute π_n, the long-term probability that exactly n tables are occupied. How much business will a restaurant lose? How many tables are occupied on average?
9. Find the line of best fit through the following data, and compute R2.

x_i	0	1	2
y_i	1	3	3

10. We wish to fit the equation
$$y = \frac{a}{x^2 + b}$$
to the following data:

x_i	1	2	0
y_i	1	1/2	1/2

(a) Transform this problem in such a way that will allow you to determine the coefficients a, b by solving a certain 2x2 linear system. Write down the linear system that you need to solve to determine a, b.

(b) Solve the system and determine a, b.

11. Suppose that you want to fit a function of the form $y = ax^p$ to some data. Transform this problem in such a way that will allow you to determine the coefficients a, p by solving a certain 2x2 linear system. Write down the linear system that you need to solve to determine a, b.