MATH 2400 HW 6

Due date: 7 Dec (wed)

1. Use the multi-variable Newton's method to find at least one root of the following system of equations:

$$x^2 + y^2 = 1;\ x = \sin(y).$$

Note: Please use the multi-variable Newton's method, not simply solving $\sin^2 y + y^2 = 1$. Starting with $x = 1, y = 1$, output the first five iterations of the method.

2. Find the line of best fit through the points $(0, 0),\ (0, 1),\ (1, 1)$.

3. The following table lists Canadian population (source: wikipedia)

Year	Population. (in millions)
1951	14.0
1961	18.2
1971	22.0
1981	24.8
1991	28.0
2001	31.0

Assuming the exponential population growth model,

$$y = ae^{bt},$$

determine a, b using the least-squares method. Plot the data along with your best-fit curve on the same graph. Using this method, estimate the current Canadian population. How does this compare with Wikipedia estimation of 34.6 million for the population in 2011?

4. The following data lists the number of students in a calculus class that obtained a given grade, from 1 to 10.

grade	1	2	3	4	5	6	7	8	9	10
# *students*	2	3	4	4	12	23	29	25	14	2

   (a) Use nonlinear fitting to fit a function of the form $y = a\exp(-b\,(x-c)^2)$ to this data. What do you get for a, b, c? Note: you need to make an educated guess about initial values of a, b, c. On the same graph, plot the points of this table as well as the fitted graph.

   (b) [BONUS] Repeat question (a), but using *linear* fitting, i.e. without having to perform the Newton iterations. On the same graph, sketch the answer you get to (a) and (b). Hint: take logs.

5. A certain country has 5 towns of equal population size, whose coordinates in the (x_i, y_i) plane are given in the following table:

x_i	0	1	2	0	2
y_i	0	0	0	1	2

As the country's benevolent dictator, you decide to build a sports palace that will service all five towns. You will also build five straight roads from the sports palace to all of the five towns. Where should the palace be located in order to minimize the cost of building the roads? That is, you want to find (x, y) to minimize $\sum_i \sqrt{(x - x_i)^2 + (x - y_i)^2}$.