1. Solve $u_t = u_{xx}$ for $x > 0$ subject to the initial condition $u(x,0) = x^2$ and a boundary condition $u(0,t) = 0$.

2. The purpose of this question is to solve $u_t = u_{xx}$ subject to initial condition $u(x,0) = x$ for $x > 0$ and a boundary condition $u_x - 2u = 0$ at $x = 0$.

   (a) Define the function
   $$f(x) = \begin{cases} x, & x > 0 \\ x + 1 - e^{2x}, & x < 0 \end{cases}.$$
   Verify that $f' - 2f$ is an odd function. Then show that $f(x)$ is the unique function such that $f(x) = x$ for $x > 0$, f is continuous, and $f' - 2f$ is odd (for $x \neq 0$).

   (b) Let v solve $v_t = v_{xx}$ subject to initial condition $v(x,0) = f(x)$ for $x \in \mathbb{R}$. Prove that $v(x,t) = u(x,t)$ whenever $x > 0$. Hint: Define $w = v_x - 2v$ and show that $w(0,t) = 0$.

   (c) Write out the solution for u, in terms of certain integrals.

3. Section 3.2, question 1.

4. Section 3.2, question 2, but please take $a, c = 1$.

5. Section 3.2 questions 3 and 4. In both cases, illustrate your solution by taking $c = 1$, $f(x) = \begin{cases} 1, & \text{if } x \in [1,2] \\ 0 & \text{otherwise} \end{cases}$ and sketching the solution at times $t = 0...3$ with increments of 0.5

6. Section 2.2, question 4

7. Section 2.3, question 6.

8. Section 2.3, question 8.