2) $u_{tt} + u = \varepsilon u^2$

Expand: $u = u(t, \tau, s)$, $\tau = \varepsilon t$, $s = \varepsilon^2 t$

$u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2$

$u_0 = A e^{it} + c.c.$

$u_{1tt} + u_1 = u_0^2 - 2u_{0t\tau}$

$= A^2 e^{2it} + \bar{A}^2 e^{-2it} + 2A\bar{A} - 2iA_\tau e^{it} + 2i\bar{A}_\tau e^{-it}$

Resonance: $A_\tau = 0$

$= A^2 e^{2it} + \bar{A} e^{-2it} + 2A\bar{A}$

$\rightarrow u_1 = \frac{A^2}{-3} e^{2it} + c.c. + 2A\bar{A}$

$u_{2tt} + u_2 = 2u_0 u_1 - 2(u_{0ts}) \rightarrow A_s i e^{it} + c.c.$

$= e^{it}\left[\frac{-\bar{A}\bar{A}}{3} \cdot 2 + 4A\bar{A}A - 2iA_s\right] + \ldots = 0$

$\rightarrow \boxed{iA_s = \frac{5}{3}|A|^2 A}$

$A = Re^{i\varphi} \rightarrow -\varphi_s R + iR_s = \frac{5}{3}R^3$

$\boxed{\varphi_s = -\frac{5}{3}R^2, \quad R_s = 0}$

$u_0 = Re^{i(\varphi + t)} + c.c. = 2R\cos\left(t + \frac{-5\varepsilon^2}{3}R^2\right)$

$U(0) = 1, \quad U'(0) = 0 \rightarrow R = 1$

$$\Rightarrow \boxed{U_0 = \cos\left(t - \frac{5}{3}\varepsilon^2 t\right)}$$