

Homework 1

1. Sketch the parametric curve $x = \sin(t)$, $y = -t$, $t = -\pi \dots \pi$. Use arrows to indicate the direction of increasing t .
2. Find a parametric equation for a circle of radius 2, centered at $(3, 4)$.
3. (a) Sketch the region enclosed by the curve $x = t^2$, $y = \sin t$, $0 \leq t \leq 1$, the x-axis and the line $x = 1$. (b) Find the area of this region.
4. (a) Sketch the region enclosed by the curve $x = t^2 - t$, $y = \sqrt{t}$, and the y-axis. (b) Find the area of this region.
5. Consider a parametric curve given by $x = \sin t$, $y = \cos^2 t$; $t \in [0, \pi/2]$.
 - (a) Sketch this curve.
 - (b) Determine the equation of the line tangent to this curve at $t = \pi/2$.
 - (c) Determine the area bounded by this curve and the x and y axes.
6. Sketch the following curves given in polar coordinates: (a) $r = 1 - \cos \theta$, $0 \leq \theta \leq 2\pi$; (b) $r = 1 + \cos(\theta)$, $0 \leq \theta \leq \pi$; (c) $r = 1 + \cos(2\theta)$, $0 \leq \theta \leq \pi$.
7. Find the length of the curve $x = e^{-t} \cos(2t)$, $y = e^{-t} \sin(2t)$, $t = 0..2$.
8. Sketch the curve $(x^2 + y^2)^3 = 4x^2y^2$ by first converting it to polar coordinates. Hint: $2 \cos \theta \sin \theta = \sin(2\theta)$.
9. A cow is tied to a silo with radius 1 by a rope just long enough to reach the opposite side of the silo. Find the area available for grazing by the cow.

