HW 5

1) Sketch the streamlines corresponding to the following potentials. In each case, indicate stagnation points (if any) and field direction.

a) $G(z) = \ln(z+1) - \ln(z-1)$

b) $G(z) = i\ln(z+1) - i\ln(z-1)$

c) $G(z) = i\ln(z+1) + i\ln(z-1)$

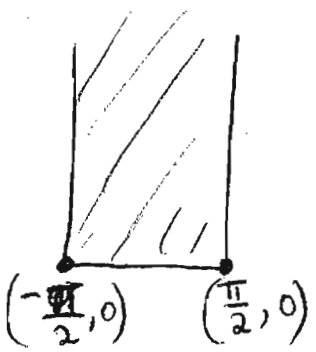
2) Show that the streamlines in (1a) and (1b) are circles and a line.

3) Consider the flow $G(z) = z + a\ln(z+i) + a\ln(z-i)$

a) Sketch streamlines for $a<0$, $a=0$, $0<a<1$, $a=1$ and $a>1$.

In each case, indicate stagnation points.

4) a) Determine the image of the domain D below under the map $w = \sin z$.



4b) Sketch the streamlines of potential

$$G(z) = \sin z$$

inside the domain D of question (4a).
[first, show that D is a streamline of G].

4c) Suppose a source is located on the boundary of D at $z_0 = \frac{-\pi}{2} + i$, of strength Q.

Determine the potential corresponding to this.

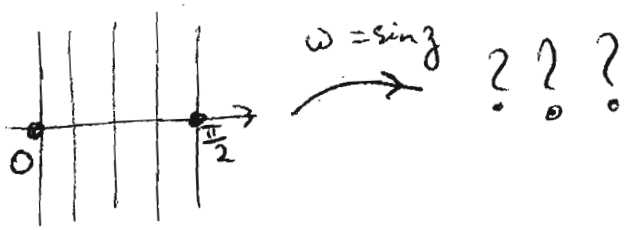
Sketch streamlines and indicate stagnation points

5) a) Sketch the images of vertical lines

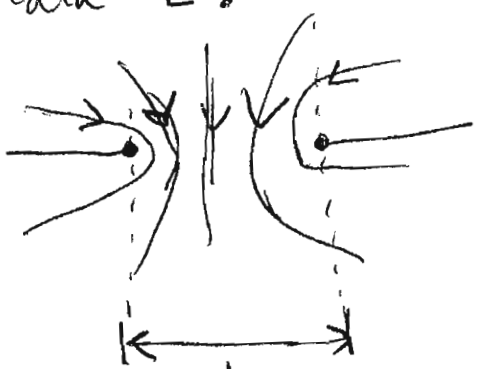
$$z = x + iy \quad , \quad x = x_0 \in (0, \tfrac{\pi}{2}) \quad , \quad y \in \mathbb{R}$$

under the map $\omega = \sin z$.

Hint: $\begin{cases} \sin(x+iy) = \sin x \cosh y + i \cos x \sinh y \ ; \\ -\sinh^2 y + \cosh^2 y = 1. \end{cases}$



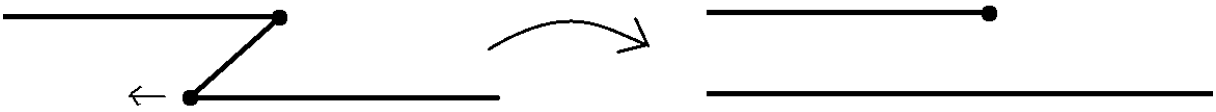
6) Find a potential for a flow through a narrow channel of width L:



What are the streamlines?

6)

(a) Use Schwarz–Christoffel transform to find the analytic mapping of the upper half-plane into the domain above the "Z"-type boundary as shown on the left hand side of this image. Note: only write $f'(z)$ down.



(b) Take the limit as indicated on the left side of the figure above to get the domain shown on the right side of the figure; compute the corresponding $f(z)$ explicitly in this case.

(c) Use a computer to sketch the streamlines for the domain on the right hand side.