Midterm

1. Use contour integration to evaluate the following integrals.

$$(a) \quad \int_0^{2\pi} \frac{1}{2 + \cos x} dx \qquad (b) \quad \int_0^{\infty} \frac{x^p}{1 + x^2}, \quad 0 \leq p \leq 1.$$

2. Solve $\Delta T = 0$ inside the region which consists of all $\mathbb{C}$ minus two circles centered at ± 2 and having radius 1. Assume $T = T_1$ on the boundary of the left circle and $T = T_2$ on the boundary of the right circle. Sketch contour plot of your solution.

3.

   (a) Find numbers R and r such that all 4 roots of the polynomial $x^4 + x + 1 = 0$ lie inside the region $r < |z| < R$.

   (b) BONUS: Find a number R such that all n roots of the polynomial $x^n + x^n a_{n-1} + \ldots + a_0 = 0$ lie inside the region $|z| < R$.

4.

   (a) For the domain as shown on the blackboard, determine the derivative $f'(z)$ of a conformal map $f(z)$ which maps the upper half-plane into this domain. Sketch the corresponding steamlines.

   (b) Using part (a), find $f(z)$ which streamlines the domain as shown. Also indicate the coordinates of the points as shown in the blackboard. Sketch the corresponding streamlines.