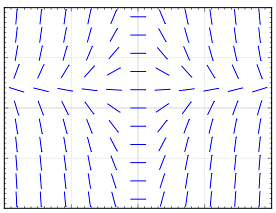
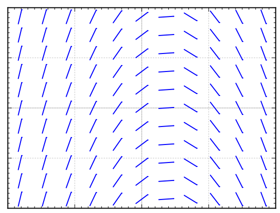
MATH 2120, Homework 1

1. (§1.1) Solve $\frac{dy}{dx} = x^2 + 1$ subject to initial condition $y(1) = 2$. What is $y(3)$?
2. (**1.1.7**) Solve $\frac{dy}{dx} = \frac{1}{y+1}$ subject to $y(0) = 0$.
3. (**1.2.6**) Match equations (1) $y' = 1 - x$, (2) $y' = x - 2y$, (3) $y' = x(1-y)$ to slope fields below. Justify.

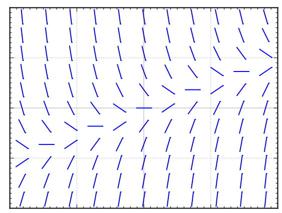
a)



b)



c)



4. (§1.2) (a) Make a rough sketch of slope field for the ODE $y' + y^3 = 0$.

   (b) Solve the problem
   $$y' + y^3 = 0, \quad y(0) = y_0$$
   and determine how the interval in which the solution exists depends on the initial value y_0.
5. (§1.2) (a) Verify that both $y_1(t) = 1 - t$ and $y_2(t) = -t^2/4$ are solutions of the initial value problem
   $$y' = \frac{-t + (t^2 + 4y)^{1/2}}{2}, \quad y(2) = -1.$$
   Where are these solutions valid?

   (b) Explain why the existence of two different solutions of the given problem does not contradict the uniqueness part of Theorem 1.2.1.
6. (§1.3) (a) Find the general solution to the ODE $y' + y^2 \sin x = 0$.

   (b) Solve $y' + y^2 \sin x = 0$ subject to initial condition $y(0) = 2$.
7. (§1.3) Consider the problem
   $$\frac{dy}{dx} = \frac{1 - 2x}{y}, \quad y(1) = -2.$$
   (a) Find the solution in explicit form.

   (b) Plot the graph of the solution.

   (c) Determine the interval in which the solution is defined.
8. (§1.3) Find (an implicit solution of)
   $$y' = \left(1 + 3x^2\right)/(3y^2 - 6y), \quad y(0) = 1.$$
9. (§1.4) Solve $y' - y = e^{2t}$ subject to $y(0) = 1$.
10. (§1.4) Solve $t\frac{dy}{dt} + y = 1$, $y(1) = 2$.
11. (§1.4) Find the value of y_0 for which the solution of the initial value problem
    $$y' - y = 1 + 3\sin t, \quad y(0) = y_0$$
    remains finite as $t \to \infty$ (i.e., uniformly bounded by some constant for all t).

12. (§1.4) A tank initially contains 120 L of pure water. A mixture containing a concentration of γ g/L of salt enters the tank at a rate of 2 L/min, and the well-stirred mixture leaves the tank at the same rate. Find an expression in terms of γ for the amount of salt in the tank at any time t. Also find the limiting amount of salt in the tank as $t \to \infty$.