HW3

1) A damped wave equation is given by

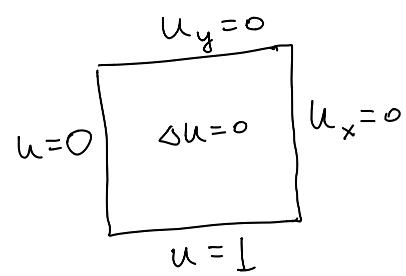
$$u_{tt} + \alpha u_t = u_{xx}$$

where $\alpha > 0$ is friction coefficient.

(a) Solve it, subject to B.C. $u(0,t) = 0 = u(\pi, t)$
and i.c. $u(x,0) = \sin(x)$.

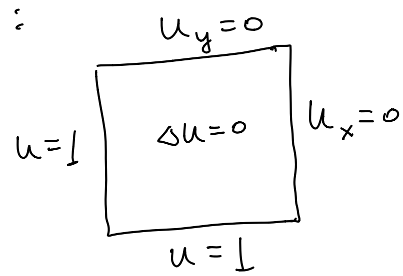
(b) Show that $u(x,t) \to 0$ as $t \to \infty$

2) (a) Solve Laplace's equation on a square $[0,1] \times [0,1]$ subject to the following B.C.



Estimate $u(\frac{1}{2}, \frac{1}{2})$ to 3 significant digits.

(b) Same as part (a) except BC:



(c) Explain what you observed.

3) (a) Read § 5.3.2.

(b) Solve the following heat eq'n with time-periodic forcing: $u_t = u_{xx} + \sin(\omega t)$.

4) Solve the heat eq'n $u_t = u_{xx}$ subject to inhomogeneous

B.C. $\begin{cases} u(0,t)=0 \\ u(1,t)=1 \end{cases}$ and I.C. $u(x,0)=f(x)$.

5) Solve the heat eq'n $u_t = u_{xx}$ subject to inhomogeneous

B.C. $\begin{cases} u_x(0,t)=0 \\ u(1,t)=1 \end{cases}$ and I.C. $u(x,0)=f(x)$.

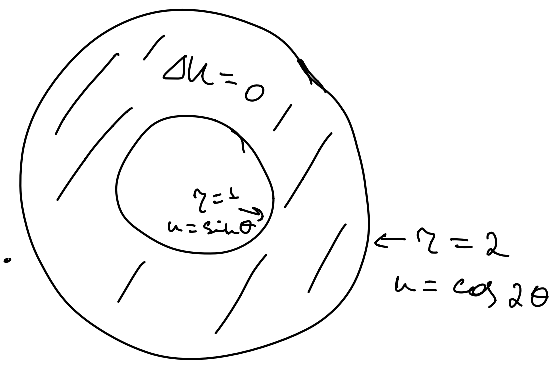
Extra Questions: (At least one of these will be on midterm)

6) Solve $\Delta u = 0$ inside the annulus $1<r<2$

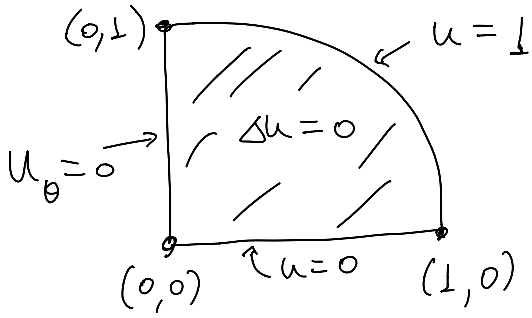
with

$u = \cos(\theta)$ when $r=1$

$u = \cos(2\theta)$ when $r=2$.



7) Solve $\Delta u=0$ on quarter-unit disk with the following B.C.



8) Solve $9u_{xx} + u_{yy} = \sin(2\pi x) + \sin(2\pi y)$

inside unit square : $x, y \in (0,1)$

and with $u=0$ on Boundaries :

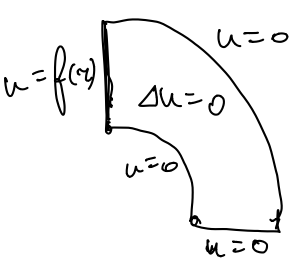
$u=0$ if $x=0$ or $x=1$ or $y=0$ or $y=1$

9) [Harder] Solve $\Delta u = 0$ on a sector

$\theta \in (0, \frac{\pi}{2})$, $r \in (1,2)$ with

B.C. $u=0$ when $r=1$, $r=2$, $\theta=0$

and $u=f(r)$ when $\theta = \frac{\pi}{2}$.



10) Solve the following problem inside unit square: