Homework questions on NLEP problem.

1. Consider the problem

$$u_t = \varepsilon^2 u_{xx} - u + \frac{u^2}{\int u^p}, \quad x \in (-1,1), \quad u'(\pm 1, t) = 0$$

   (a) Construct the steady state in the form of a single spike centered at the origin.

   (b) Formulate a corresponding nonlocal eigenvalue problem for the large eigenvalue. Is it stable?

2. The Gray-Scott model is given by

$$\begin{aligned} u_t &= \varepsilon^2 u_{xx} - u + vu^2; \\ \tau v_t &= D v_{xx} - v + 1 - vu^2 \frac{A}{\varepsilon}. \end{aligned}$$

   For this question, suppose that

$$A = O(1), \quad \varepsilon \ll 1, \ \tau = 0.$$

   (a) Find the spike-type solution on the domain $[-L, L]$ with Neumann boundary conditions. Note that you should obtain two different solutions corresponding to two different roots of a certain quadratic equation. Find a number $A_c > 0$ such that the two solutions you found exist if and only if $A > A_c$.

   (b) Formulate the nonlocal eigenvalue problem that determines the stability of the solutions found in part (a). Show that one of the solutions you found in part (a) is unstable while the other is stable.