Introduction: Dissipative localized structures in extended systems

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Localized structures belong to the class of dissipative structures found far from equilibrium. Contributions from the most representative groups working on a various fields of natural science such as biology, chemistry, plant ecology, mathematics, optics, and laser physics are presented. The aim of this issue is to gather specialists from these fields towards a cross-fertilization among these active areas of research and thereby to present an overview of the state of art in the formation and the characterization of dissipative localized structures. Nonlinear optics and laser physics have an important part in this issue because of potential applications in information technology. In particular, localized structures could be used as "bits" for parallel information storage and processing.
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This Focus Issue is devoted to recent progress, new ideas, and novel techniques related to the study of localized structures (LSs) in spatially extended systems. It gathers contributions from various fields of nonlinear science: biology, chemistry, ecology, mathematics, laser physics, and optics. Many systems in nature can support LSs. Examples include laser-generated pulses in optical cavities, animal skin patterns, vegetation patches in arid and semiarid landscapes, localized chemical spots, metal corrosion, hot spots in microwave heating, and cell aggregation in chemotaxis.

Localized structures are nonlinear inhomogeneous states of spatially extended systems that offer short spatial range correlations in comparison to long-range correlations characteristics of patterns (periodic states). Their main feature is to allow the confinement of energy, chemical concentration, or phytomass density. In optics, for example, light confinement via LSs can have a broad range of applications in spectroscopy, communications, information storage and processing, astronomy, medicine, and biology. In practice, localized structures appear as spikes, interfaces, spots, pulses, dissipative solitons, cavity solitons, or autosolitons. They can be either stationary or time dependent. In the latter case, many phenomena have been observed, such as spike motion, oscillating solitons, and complex two-dimensional patterns. Despite the diversity of situations and contexts in which they are found, these examples of localized structures share common underlying causes. Mathematically, LSs can be viewed as homoclinic or heteroclinic orbits of an underlying dynamical system. Generally speaking, dissipative structures arise from to the balance between a positive feedback mechanism (chemical reactions, light-matter interaction) that tends to amplify spatial inhomogeneities, and a transport process such as diffusion, thermal diffusivity or diffraction, which on the contrary tends to restore spatial uniformity.

The spontaneous emergence of spatial patterns out of the homogeneous steady state goes back to the pioneering work by Alan Turing within the context of morphogenesis. Turing's instability is one of the few universally applicable mechanism leading to patterns that are intrinsic. The wavelength is determined by the chemical or physical parameters and not by the boundary conditions. Not all dissipative structures share this property. In some reaction-diffusion problems, the wavelength is proportional to the domain size. Localized structures are usually excited in the pinning region involving the homogeneous steady state and the periodic dissipative structures. Therefore, the occurrence of a subcritical Turing bifurcation is often the prerequisite condition for the emergence of LSs. Note, however, that there exist other classes of LSs that can be stable far from any Turing bifurcation. By now, a large body of literature exists on the study of localized structures in biology, chemistry, physics, and mathematics.

In the last two decades, considerable progress has been made towards the understanding of these structures. With the advent of fast computers, many numerical simulations have confirmed the presence of localized patterns in partial differential equations models in one or more dimensions. Moreover, LSs have been experimentally observed in several chemical, optical, and plant ecological systems, and in material science.

The purpose of this Focus Issue is to bring together researchers working in all aspects of LS: formation, characteristics, dynamics, stability, and interaction. We hope that it will lead to a better understanding of this topic and will bring a cross-fertilization among different fields of nonlinear science that evolved more or less independently. The papers in this issue are grouped by area. Papers 1–5 describe various theoretical aspects of localized pattern formation. Papers 6–9 are devoted to nonlinear chemical and biological systems. Finally, Papers 10–18 describe LSs and patterns in nonlinear optics.

Burke and Knobloch[1] describe the homoclinic snaking behavior that occurs in the pinning region. They use the paradigmatic Swift-Hohenberg equation and exploit its variational structure to discuss the wavelength of LSs. They also examine stability in a two-dimensional system. For a large class of nonequilibrium systems, a minimal mathematical model, i.e., a nonvariational Swift-Hohenberg equation, is derived by Kozyreff and Tlidi.[2] Nonvariational effects preclude the existence of a Lyapunov functional to minimize and may therefore be important for the stability of LSs. Nishiura *et al.*[3] examine the interaction of a traveling pulse with a spatial inhomogeneity. A number of different scenarios (pulse splitting, reflection, etc.) are described. A reduced equation that captures the motion of the pulse near the inhomogeneity is derived. The stability of asymmetric spike patterns in the classical Gierer-Meinhardt model is examined by Iron and Rumsey.[4] A complete discussion of stability near the point of bifurcation when the asymmetric spikes first emerge from the symmetric branch is provided. Maini *et al.*[5] perform a stability analysis of Gierer-Meinhardt system with mixed boundary conditions. A new type of instability is revealed for certain parameter values.

Halloy *et al.*[6] report on the existence and stability of stable standing-wave patterns in coupled complex Ginzburg-Landau equations, subjected to parametric forcing. A model of chemotaxis that incorporates the elastic properties of cells is presented by Wang and Hillen.[7] The resulting model is studied analytically and numerically. The authors report novel spike dynamics such as spike insertion. In the context of plant ecology, Meron *et al.*[8] study vegetation patches in arid and semiarid landscapes, which are interpreted as localized structures of biomass and water. Spot and stripe patterns are reported together with a complex bifurcation structure. The paper by Vanag and Epstein[9] reviews experimental and theoretical investigations of localized structures such as stationary spots, oscillons, clusters, and moving and breathing spots in chemical reaction-diffusion systems.

Brazhnyi *et al.*[10] present the existence and stability of dissipative localized structures in periodically modulated quadratic media. Localized modes are formed by pump-signal pairs whose phase difference is a crucial parameter for their existence and stability. Akhmediev *et al.*[11] perform an extensive study of the full $(3+1)$-dimensional spatiotemporal stable optical solitons. Stationary solutions, rotating, and time-dependent bound states are reported. An experimental technique to control position and motion of localized structures in a single feedback liquid crystal light valve system is presented by Gütlich *et al.*[12] The interaction of dissipative optical solitons in active nonlinear fibers with Bragg grating is addressed by Rosanov and Tran.[13] New phenomena are studied, including the discreteness of moving solitons velocity. The discrepancy between theoretical predictions and experiments in the region of existence of LS is discussed by Firth *et al.*[14] A nonlocal nonlinearity is suggested to resolve this discrepancy. More insight on this crucial and timely question can also be gained from investigations on snaking bifurcations reported by Burke and Knobloch.[1] A study of Turing (often called modulational) instabilities in a nonlinear resonator filled with a slice of left-handed materials, i.e., a material with a negative refraction index, with either Kerr or quadratic nonlinearities is presented by Tassin *et al.*[15] A review on solitary pulses in linearly coupled complex Ginzburg-Landau equations that have a wide spectrum of applications is presented by Malomed.[16] In particular, it provides a good insight on pulse dynamics in optical fibers. Pomeau and Le Berre[17] discuss an interesting new issue of the tunneling of quantum localized structures in a twin-core nonlinear optical fiber. Finally, Brambilla *et al.*[18] discuss recent developments and progress on localized structure formation in a semiconductor based on quantum dots.

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