The Q-switching instability in passively mode-locked lasers

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Joint work with

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Haus master equation

$$E_T = \left(N - 1 - \frac{a}{1 + b|E|^2}\right) E + E_{\theta\theta}$$
$$N_T = \gamma \left[A - N - NL^{-1} \int_0^L |E|^2 d\theta\right].$$

- Models a passively mode-locked laser in a cavity with a fast saturable absorber.

- Supports the formation of a steady pulse

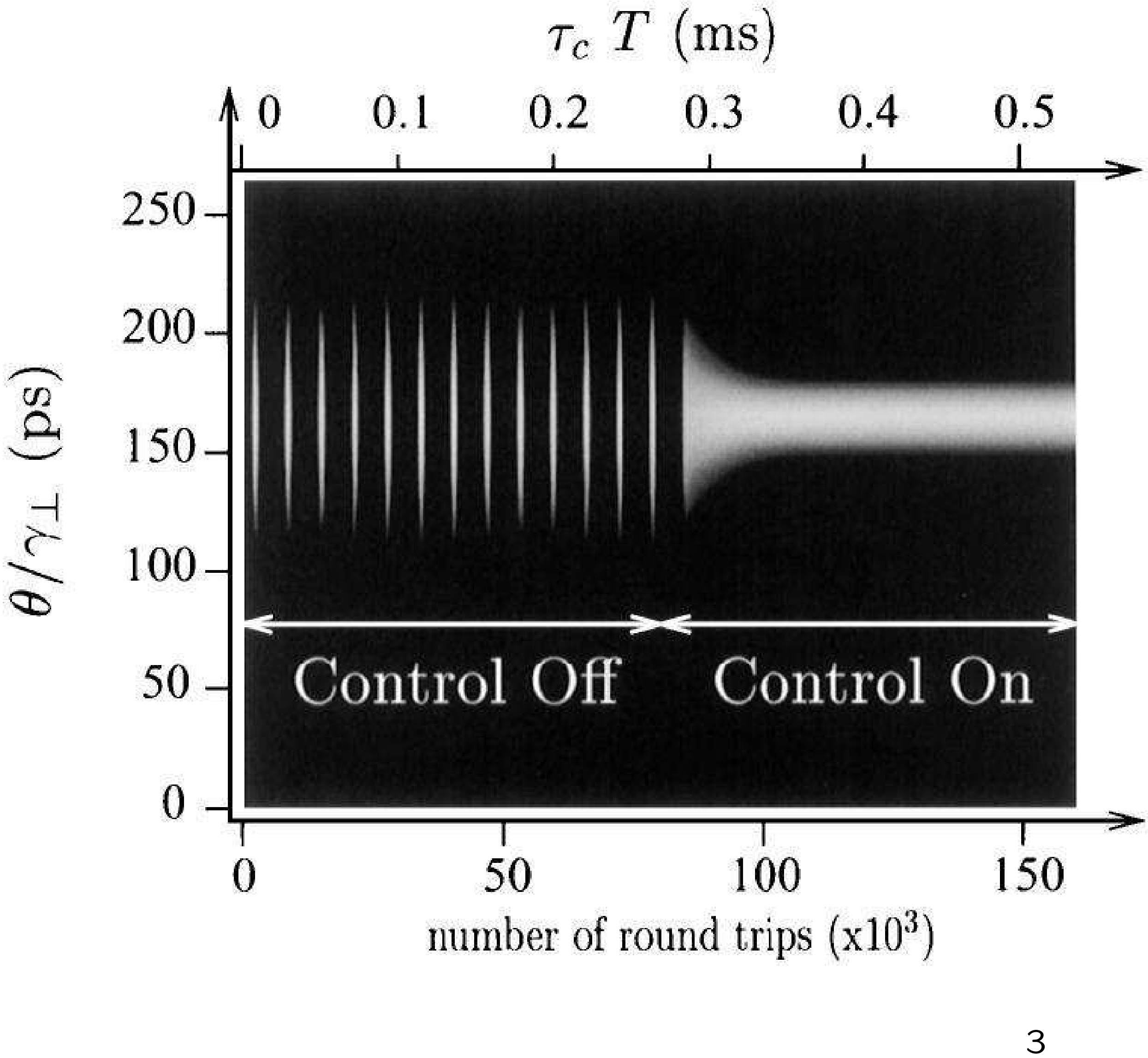
- Numerical experiments of Joly and Bielawski (2001) show a Hopf bifurcation.

Joly and Bielawski (2001)

Parameters:

$$A = 8, L = 3800, \gamma = 1.4\text{e-}4, b = 0.01.$$

Profile oscillations was found with $a \geq 0.003$.



Scaling

$$
\begin{aligned}
N &= 1+\omega x,\ E=(A-1)^{1/2}y, \\
z &= \theta/L \text{ and } s=\omega T
\end{aligned}
$$

where

$$\omega \equiv \sqrt{2\gamma(A-1)}$$

$$
\begin{aligned}
x_s &= \frac{1}{2}\left[1-\frac{\omega}{A-1}x-(1+\omega x)\int_0^1 y^2 dz\right], \\
y_s &= (x-\alpha\frac{1}{1+\beta y^2})y+Dy_{zz}
\end{aligned}
$$

where

$$\alpha \equiv \frac{a}{\omega},\ \beta \equiv b(A-1) \text{ and } D \equiv \frac{1}{\omega L^2}.$$

is known as the laser relaxation oscillation frequency.

Steady state

Expand in ω^2, $y = y_0 + \omega^2 y_1 + \cdots$, $x = x_0 + \omega^2 x_1 + \cdots$,

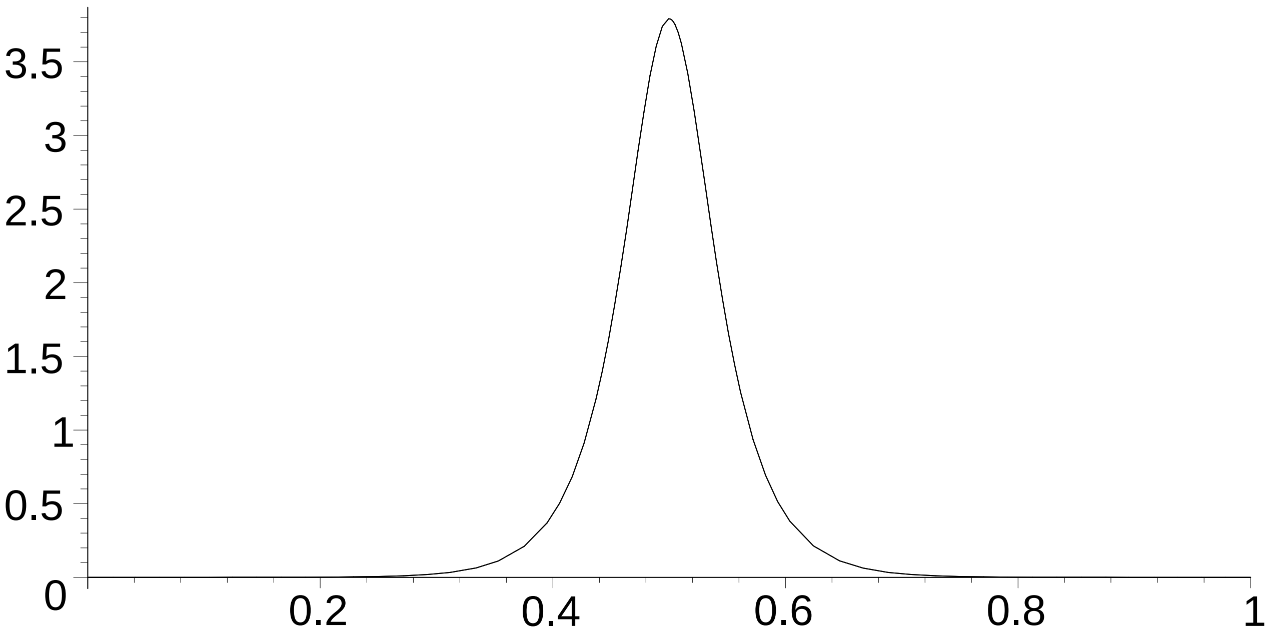
$$0 = (x_0 - \alpha \frac{1}{1+\beta y_0^2}) y_0 + D y_{0zz},$$
$$0 = 1 - \int_0^1 y_0^2 dz.$$

which has the form $u'' = F(u)$ from where,

$$x_0 = \alpha \frac{\ln(1+\beta y_M^2)}{\beta y_M^2}.$$

where y_M is the maximum point of the spike. Then use the second equation to solve for y_M and x_0 simultaneously.

Typical solution for y_0:



Hopf bifurcation

- Linearize around the steady state,

$$y \sim y_{st}(z) + e^{\lambda s}\phi(z)$$

To leading order, we find $\lambda \sim i$. Near Hopf bifurcation we expand

$$\lambda = i + \omega\lambda_1 + \cdots$$

Imposing $\mathrm{Re}\lambda_1 = 0$ yield

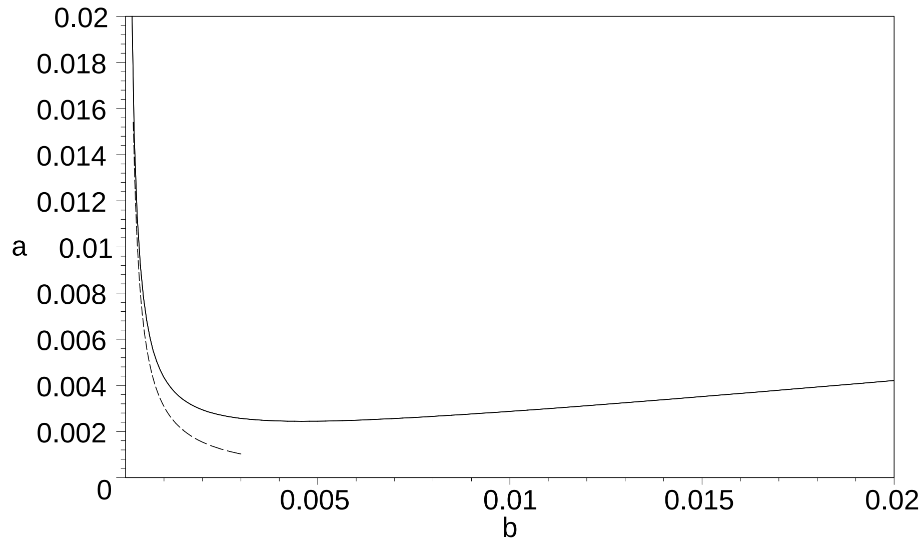
$$\boxed{\frac{A\omega}{A-1} - 16\beta D \int_0^\infty y_0^2 d\zeta \int_0^\infty \left(\frac{y_0^2}{1+\beta y_0^2}\right)^2 d\zeta = 0.}$$

• We predict $a_h = 0.0029$, (compared to 0.003 numerically).

• Simple explicit formula is possible when b is small:

$$a_h \sim \frac{\sqrt{6\gamma A}}{(A-1)Lb}.$$

• Otherwise, a_h is found by numerically evaluating some integrals.



Bursting behaviour

Generalization: slow absorber