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Editorial

Emergent behaviour in multi-particle systems with non-local interactions

1. Introduction

This special issue grew out of a BIRS workshop of the same title that was held in February 2012. It contains new research contributions from a broad spectrum of researchers on topics related to emergent behaviour. The goal is to present the current research in a single volume to showcase the diversity and vitality of this area and to serve as a useful resource for future reference.

Collective group behaviour is a fascinating natural phenomenon that is observed at all levels of the animal kingdom, from beautiful bacterial colonies, insect swarms, fish schools and flocks of birds, to complex human population patterns. The emergence of very complex behaviour is often a consequence of individuals following very simple rules, without any external coordination. In recent years, many models of group behaviour have been proposed that involve nonlocal interactions between the species [1–4]. Related models also arise in many physical systems such as granular media [5–8], self-assembly of nanoparticles [9,10], vortex dynamics in Bose-Einstein Condensates [11–14] and other media [15,16], n-body problems [17], synchronization in biological systems [18–20] and molecular dynamics simulations of matter [21].

Due to their nonlocal nature, these systems can exhibit complex and novel phenomena that pose challenging questions and motivate the development of new mathematical techniques. They typically lead to coherent and synchronized structures apparently produced without the active role of a leader. The instantaneous emergence has been called self-organization [22,23], and has been observed across a wide range of species, even for some microorganisms such as myxobacteria [24].

Most mathematical models of self-organization are based on discrete systems [1–3,25,19] incorporating certain effects that we might call the "first principles" of swarming. Animals are typically modeled as simple particles following certain microscopic rules determined by their position and velocity inside the group and by the local density of animals. These first principles are based on modelling the "sociological behavior" of animals with very simple rules such as the social tendency to produce grouping (attraction/aggregation), the inherent minimal space they need to move without problems and feel comfortably inside the group (repulsion/collisional avoidance) and the mimetic adaptation or synchronization to a group (orientation/alignment). Models based on these principles are classical in fish modelling [26,27]. Even if these minimal models contain very basic rules, the patterns observed in their simulation and their complex asymptotic behavior is already very challenging from the mathematical viewpoint.

The source of tendency to aggregate can also be related to factors other than sociological, such as survival fitness of grouping against predators, collaborative effort in food finding, etc. Moreover, one can incorporate a variety of interaction mechanisms between animals such as those produced by certain chemicals, pheromone trails for ants, the interest of the group to stay close to their roost, physics of swimming/flying, etc. Although the minimal models based on "first principles" are quite rich in complexity, it is interesting to incorporate more effects to render them more realistic as for instance described in [23,28–30].

Several micro and macroscopic models developed in recent years have attracted attention of many mathematicians, in particular kinetic aggregation models [31–34], second order models for self-propelled particles with attraction and repulsion effects [35–37], and Cucker–Smale model of aligment [38]. For instance, the authors in [36] use the ideas of H-stability from statistical mechanics to classify various pattern morphologies that appear for different parameter values, including translational flocks, rotating single and double mills, rings and clumps. On the other hand, in the simpler alignment models [38], we get generically a flocking behavior. Much more elaborate models starting from these basic building blocks are capable of simulating collective behaviors in systems with a large number of agents N. Control of large agent systems is important not only for the somehow bucolic example of understanding animal behavior, but also for pure control engineering for robots and devices with the aim of unmanned vehicle operation and their coordination, see [39,40] and the references therein.

When the number of agents is large, the use of continuum models for the evolution of a density of individuals becomes essential. Some continuum models were derived phenomenologically [31,41,42] including attraction–repulsion mechanisms through a mean force and spatial diffusion to deal with the anti-crowding tendency. Other continuum models are based on hydrodynamic descriptions [43,44,14] derived by means of studying the fluctuations or the mean-field particle limits. Hyperbolic systems have also been proposed [45–47]. The essence of the kinetic modelling is that it does connect the microscopic world, expressed in terms of particle models, to the macroscopic one, written in terms of continuum mechanical systems. A very recent trend of research has been launched in this direction in the last few years, see for instance [48,49,44,50–54] for different kinetic models in swarming. Introducing noise in these models can lead to phase transitions, a line of research which is wide open [1,55–57].

Finally, variational approaches have been very fruitful to attack steady states and their stability for first order models of swarming. A very classical model in this field is the Patlak–Keller–Segel model for chemotactic cell movement [58]. Many exciting developments have happened in this direction in the last years [59–62] and these variational tools have had nice implications in the theory of first order models [63,64]. Fluid mechanics techniques have also been adapted to the aggregation equation to deeply analyse its qualitative properties [65–67].

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The common feature of these models is that they all lead to some non-locality in the equations, either in the form of a large system of ODE's with global coupling, or as a PDE with non-local kernels (integral terms). The analysis, asymptotic behavior, numerical simulation, pattern formation and their stability in many of these models still remain unexplored research territory. The development of these models has in part been motivated by increased use of computers which allows for easy experimentation. In many of these models, novel and exciting phenomena have been observed numerically. However, the fundamental understanding of observed patterns and their dynamics has been lagging. The time is ripe for development of better analytical tools which would allow to gain a better insight of these models.

2. Overview of papers in the special issues

Several major themes are represented in this special issue which we now summarize.

2.1. First-order particle models

One of the simplest models of interacting particles that yields very complex dynamics is

$$\frac{d}{dt}x_i = \frac{1}{N}\sum_{j\neq i}^{N} F(|x_i - x_j|)\frac{x_i - x_j}{|x_i - x_j|}. \tag{1}$$

where $F(r)$ models the interaction force between the particles [68]. In the hydrodynamic limit $N \to \infty$, the model yields an intergro-differential equation

$$\rho_t + \nabla_x \cdot (\rho v) = 0; \qquad v(x) = \int_{\mathbb{R}^d} F\left(|x-y|\right)\frac{x-y}{|x-y|}\rho(y)dy. \tag{2}$$

We refer to (1) or (2) as the aggregation model. This basic model is of fundamental importance as it is among the simplest non-local models that exhibits many of the phenomena that are found in more complex models. In part because of this, the aggregation model has been a very active area of research over the past decade; there are by literally hundreds of papers on this model and its variations; see for example [69,34,31,41,70] and references therein. Despite this intensive research, there are many new aspects that have recently come to light and which are addressed in this issue.

Balague, Carrillo, Laurent and Raoul [71] investigate the radially symmetric solutions of the aggregation model in dimension 2 and higher with repulsive–attractive radial potentials. Under some conditions on the potential, they show that radially symmetric solutions converge toward a spherical shell stationary state. Such steady states have been recently studied in [32,72]. They also generalize this analysis to singular stationary states supported on hypersurfaces which are not necessarily spheres. They show that dimensionality of the solution is related to the growth of the potential at the origin.

Hughes and Fellner [73] introduce several motility mechanisms to the aggregation model, including linear and nonlinear diffusion. A variety of exact analytical results are discussed, including equilibria, time-dependent solutions, and transitions between asymptotic collapse and asymptotic escape.

Fetecau and Huang [74] consider the potential which consists of short-range Newtonian repulsion and long-range power-law attraction. They use the method of moving planes to show that the unique equilibria that are supported on a compact set are radially symmetric and are monotone in the radial coordinate. The authors also study the asymptotics for several limiting cases of the exponent of the power-law attraction. Unlike related works [62,75] where exact solutions were computed for specific parameter choices, the results in this paper rely on formal asymptotics and analytical techniques.

Kolokolnikov, Huang and Pavlovski [76] study singular and near-singular patterns for certain classes of potentials. Recent studies [32,72] have demonstrated that the aggregation equation exhibits a very rich solution structure, such as steady states consisting of rings, spots, annuli, N-fold symmetries, soccer-ball patterns etc. In this work the authors show that many of these patterns can be understood as singular perturbations off lower-dimensional equilibrium states. For example, an annulus is a bifurcation from a ring; soccer-ball patterns bifurcate off solutions that consist of delta-point concentrations. Using asymptotic methods, the authors describe the form and stability of many of these patterns, including ellipses and annular-like solutions.

Yao and Bertozzi [77] study radially symmetric finite-time blow-up dynamics for the aggregation equation with degenerate diffusion in arbitrary dimension, and for power-law attractive kernels. Depending on model parameters and the initial data, they show that the solution exhibits three kinds of blow-up behavior: self-similar with no mass concentrated at the core, imploding shock solution, and near-self-similar blow-up with a fixed amount of mass concentrated at the core.

2.2. Second order models

Models of self-propelled particles typically take acceleration as well as self-propulsion of particles into account. An example of such a model is [36],

$$\frac{d}{dt}v_i = \left(\alpha - \beta|v_i|^2\right)v_i + \sum_{j\neq i} F(|x_i - x_j|)\frac{x_i - x_j}{|x_i - x_j|}; \quad \frac{d}{dt}x_i = v_i \tag{3}$$

where $F(r)$ represents the interaction force between the particles and the term $\left(\alpha - \beta\,|v_i|^2\right)v_i$ is the self-propulsion force. These models typically lead to complex dynamics including swarms, mills and double mills [36,43,44,37]. A related system is the Cucker–Smale equations modelling the flocking of birds [38]; in its simplest form it reads

$$\frac{d}{dt}v_i = \frac{\lambda}{N}\sum_{j=1}^{N}\frac{1}{\left(1+|x_i - x_j|\right)^{\beta}}\left(v_j - v_i\right); \quad \frac{d}{dt}x_i = v_i. \tag{4}$$

The contributions which we summarize below study different aspects of these models.

Carlen, Chatelin, Degond and Wennberg [78] study the propagation of chaos in biological swarm models. They consider models where pairs of particles interact to adjust their velocities one to each other. In the first process, called 'BDG', they synchronize their average velocity up to some noise. In the second process, called 'CL', one of the two particles tries to adjust its velocity to that of the other. This paper establishes the master equations and BBGKY hierarchies of these two processes. It investigates the infinite particle limit of the hierarchies at large time scales. It shows that the resulting kinetic hierarchy for the CL process does not satisfy propagation of chaos. Numerical simulations indicate that the BDG process has similar behavior to the CL process.

Carrillo, Martin and Panferov [79] consider a particle system (3). They introduce a class of interaction potentials for which macroscopic equations allow for explicit solutions in terms of special functions, and which admit flock and rotating mill states. These special interaction potentials are the two-dimensional analogues of the one-dimensional Morse potential, which also admits a solution [34]. The analytical solution is compared with full dynamical simulations of the underlying particle system and close agreement is obtained.

Vecil, Lafitte and Linares [80] perform a numerical study of the self-propelled system (3) in a 3D setting. They identify parameters that govern the possible asymptotic states for this system (clumps, spheres, dispersion, mills, rigid-body rotation, flocks). They then describe the kinetic system derived from the particle model in the limit as N tends to infinity. They propose a numerical scheme based on the kinetic system in 1D and perform a numerical analysis devoted to trying to recover asymptotic patterns similar to those emerging for the equivalent particle systems, with particles originally evolving on a circle.

2.3. Stochasticity and modeling

Most systems in nature have a random component. The presence of noise is often one of the driving forces that can completely alter the behaviour of the system. Several papers in this issue discuss how to model noise and its effect on the system.

Burger, Haškovec and Wolfram [81] examine models of biological aggregation based on randomly moving particles with individual stochasticity depending on the perceived average population density in their neighborhood. They consider both first and second-order models. Instead of having an attractive potential, the aggregation is obtained exclusively by reducing the individual stochasticity in response to higher perceived density. In the mean-field limit, these models yield nonlocal degenerate diffusion. Linear stability analysis of the continuum limit is used to identify conditions for pattern formation; well-posedness is studied. They also present results of numerical simulations for both the first- and second-order model on the individual-based and continuum levels of description.

Gazi [82] describes a model of swarm dynamics based on Lagrangian dynamics, associated with an energy formulation. The concept of biological potential energy is introduced. This flexible approach allows to easily incorporate various extensions; some examples studied include predator and environmental effects.

Galante and Levy [83] study a model of cyanobacteria. In previous works [84,85], the authors developed a stochastic particle system describing local interactions between cyanobacteria. They focused on the common freshwater cyanobacteria Synechocystis, which are coccoidal bacteria that utilize group dynamics to move toward a light source, a motion referred to as phototaxis. To gain further understanding of the group dynamics, in this paper the authors replace the stochastic model with a system of ordinary differential equations describing the evolution of particles in one dimension. Unlike many other models, the emphasis here is on particles that selectively choose one of their neighbors as the preferred direction of motion. Numerical simulations are conducted to study the stability, size, and merging of aggregations.

Rodríguez [86] studies a class of 'reaction–advection–diffusion' systems of partial differential equations, which can be taken as basic models for criminal activity. This class of systems are based on routine activity theory and other theories, such as the 'repeat and near-repeat victimization effect' and were first introduced in [87]. In these models the criminal density is advected by a velocity field that depends on a scalar field, which measures the appeal to commit a crime. The author gives several results on local and global well-posedness of solutions.

3. Discussion

This special issue gathers articles by foremost experts on the subject, and covers many recent results on modelling multi-particle systems. Mathematically, the two main approaches to study multi-particle systems consists in applying the theory of dynamical systems or by taking the continuum limit, which typically results in a PDE system that involves integral terms. Due to their nonlocal nature, these systems often lead to novel phenomena that have motivated the development of new mathematical tools and pose new problems that are further explored in this issue. The diversity of the topics involved and the backgrounds of the researchers attest to the vitality of this exciting area of research, which is currently undergoing an explosive development.

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Table of contents

Theodore Kolokolnikov
José A. Carrillo
Andrea Bertozzi
Razvan Fetecau
Mark Lewis