Math/Stat 2300 Final Review

Final Exam: Wednesday, April 21, 9am to 12pm, Dal Arena

Reminder: Please bring your student IDs, a calculator and a ruler to the exam

Material Covered:

- Modeling with difference equations
- Difference equations: equilibrium values, stability, solution
- Proportionality; Geometric Similarity
- Analytic fitting; Least squares fitting, Linear Regression
- Interpolation: Lagrange polynomials, Cubic splines
- Modeling with differential equations: population growth, separable equations, linear equations, equilibrium values, phase-line diagrams,
- Numerical approximations; Euler's method
- Graph theory: adjacency matrix, types of graph models
- Simulation: Monte Carlo simulation, types of models

Practice Questions

Please look at the midterm review problems and the midterm for practice for the material before the midterm.

The following problems give you some idea of the types of questions that might be asked on the final. Keep in mind that these questions do not necessarily cover all possible questions that you could be asked on the final.

1. Explain the idea of using cubic splines for interpolation of a set of data (x_i, y_i), $i = 1, 2, \ldots m$.
2. Consider the differential equation

$$\frac{dP}{dt} = r(M - P)P$$

   (a) What are the equilibrium value(s) for this differential equation?

   (b) What is $\frac{d^2P}{dt^2}$ in terms of P?

(c) Draw the phase-line diagram including the sign of the second derivative.

(d) Using part (c), draw a few solution curves in the P-t plane. Make note of the equilibrium values on your graph.

3. Find the solution to the initial value problem

$$\frac{dy}{dt} = (2t + 3t^2)y, \;\; y(0) = 1$$

4. Find the general solution of

$$y' + 2y = e^{-x}$$

5. (a) Using Euler's method, find the first three approximations y_1, y_2, y_3 for the initial value problem

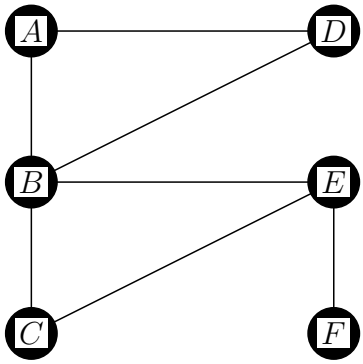
$$y' = (1 + y)x, \;\; y(0) = 1$$

with $\Delta x = 0.1$.

(b) What effect will reducing the step-size Δx have on the approximations?

6. Describe the graph associated with a particular social network. What do the nodes represent? What do the edges represent?

7. (a) Write the adjacency matrix for the following graph:



(b) Write the graph associated with the following adjacency matrix

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

8. (a) For what is Dijkstra's algorithm used?

(b) For what is Kruskal's algorithm used?

9. (a) Give an example of a problem that could be modeled using simulation.

   (b) Explain the use of Monte Carlo simulation from a probability distribution.

10. Consider a model for the long-term dining behaviour of the students at a particular college. It is found that 25% of the students who eat at the college's Grease Dining Hall return to eat there again, whereas those who eat at Sweet Dining Hall have a 93% return rate. These are the only two dining halls available on campus, and assume that all students eat at one of these dining halls.

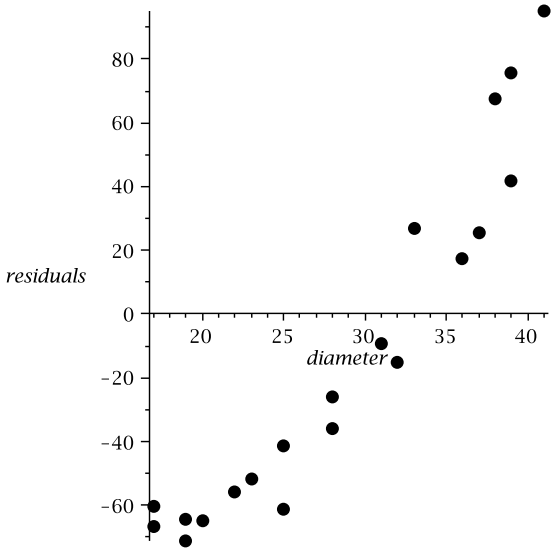
    (a) Draw the Markov Chain graph that corresponds to this problem. Be sure to define any variables you use.

    (b) What is the transition matrix for this problem?

    (c) Write a system of difference equations to model the dining behaviour. Be sure to define any variables you us.

11. (a) Why would we use linear regression?

    (b) Consider the following plot of residuals vs the independent variable x:



What do the residuals indicate about the model? What does it suggest for refining the model?