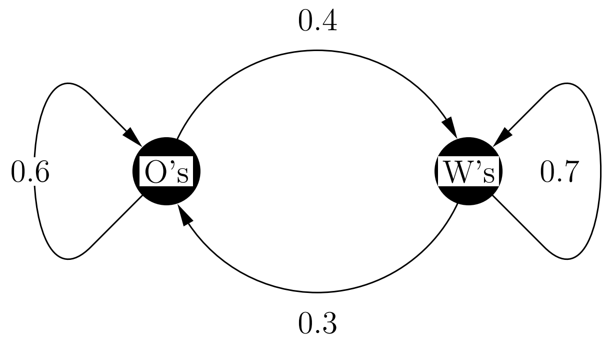
Math/Stat 2300 **Discrete Probabilistic Modeling**
from text *A First Course in Mathematical Modeling*, Giordano, Fox, Horton, Weir, 2009.

Markov Chains

A Markov chain is a process in which there are the same finite number of states or outcomes that can be occupied at any given time.

Example. Consider a rural town where the residents only purchase two brands of cereals: O's and W's.

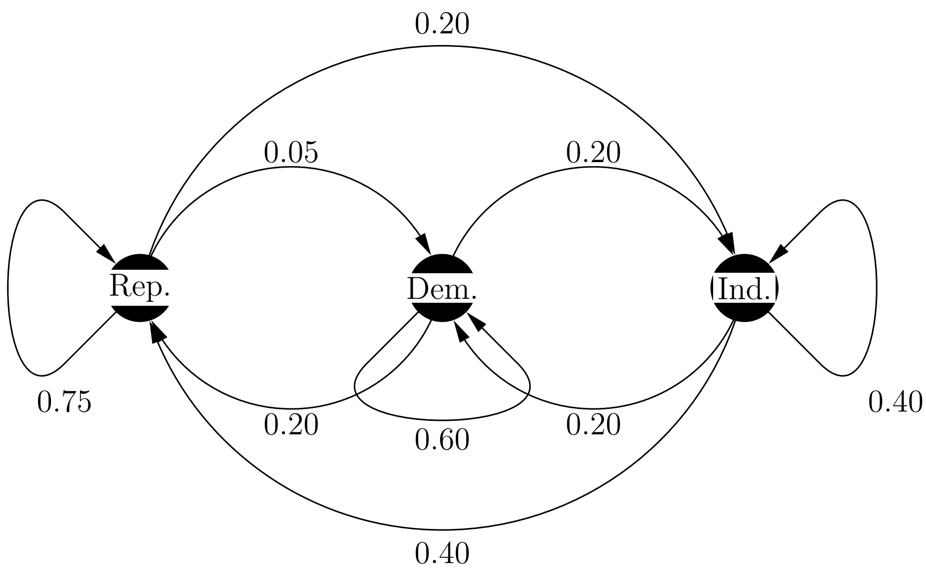


The diagram gives the probability of what a resident will purchase next.

	O's	W's
O's	0.6	0.4
W's	0.3	0.7

Example (Voting Tendencies)

		Next State		
		Republicans	Democrats	Independents
	Republicans	0.75	0.05	0.20
Present	Democrats	0.20	0.60	0.20
State	Independents	0.40	0.20	0.40



Model:
Let

$$\begin{aligned} R_n &= \text{percentage of voters to vote Republican in period } n \\ D_n &= \text{percentage of voters to vote Democratic in period } n \\ I_n &= \text{percentage of voters to vote Independent in period } n \end{aligned}$$

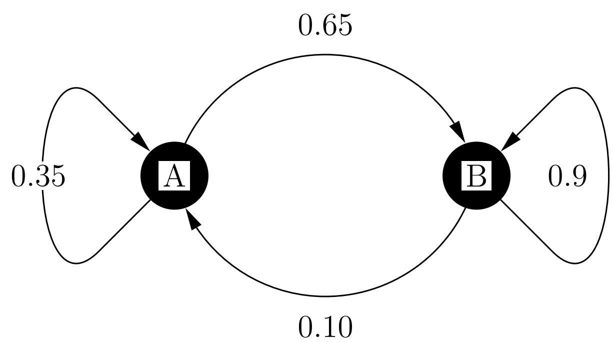
$$\begin{aligned} R_{n+1} &= 0.75R_n + 0.20D_n + 0.40I_n \\ D_{n+1} &= 0.05R_n + 0.60D_n + 0.20I_n \\ I_{n+1} &= 0.20R_n + 0.20D_n + 0.40I_n \end{aligned}$$

A **Markov Chain** is a process consisting of a sequence of events with the following properties:

- An event has a finite number of outcomes, called states. The process is always in one of these states.
- At each stage or period of the process, a particular outcome can transition from its present state to any other state or itself.
- The probability of going from one state to another in a single stage represented by a transition matrix for which entries in each row are between 0 and 1; each row sums to one.

Example.

Consider the pollution in two adjoining lakes in which the only flow is between the lakes. Let a_n and b_n be the total amounts of pollution in Lake A and B, respectively, after n years. The following graph gives the two state Markov Chain for lake pollution.



The transition matrix for this Markov chain is

	A	B
A	0.35	0.65
B	0.10	0.90

The dynamical system model:

$$\begin{aligned} a_{n+1} &= 0.35a_n + 0.1b_n \\ b_{n+1} &= 0.65a_n + 0.9b_n \end{aligned}$$