Math 2300 Assignment #1

due by 4:30pm January 26, 2010

1. (a) Find the solution to the difference equation $x_{n+1} = \frac{1}{2}x_n$, $x_0 = 2$.

   (b) Find the solution to the difference equation $x_{n+1} = \frac{1}{2}x_n + 1$, $x_0 = 2$.

2. Consider a rumor spreading through a company of N employees. This can be modelled like the spreading of a contagious disease. We have the model:

$$x_{n+1} = x_n + k(N - x_n)x_n$$

   where n is the time in days.

   (a) What does x_n represent here? What does $(N - x_n)$ represent?

   (b) What does k represent? What would be the meaning of a small k value? What would be the meaning of a large k value?

   (c) If $k = 0.001$ and there are 1000 employees in the company, how many days does it take for a rumor to spread to more than 60 people if the rumor is initially heard by 4 employees?

3. *Whale Population.* Consider the survival of a population of whales. Assume that if the number of whales falls below a minimum survival level m, then the species will become extinct. In addition, assume that the population is limited by the carrying capacity M of the environment.

   (a) Write a difference equation model to describe the whale population each year. Define all variables used. (Let k be the constant of proportionality)

   (b) Assume that $M = 5000$, $m = 100$ and $k = 0.0001$. Find the fixed points of the model. Determine the stability of these fixed points.

4. Consider the model

$$x_{n+1} = rx_n\left(1 - \frac{x_n}{K}\right), \quad r > 0, \ K > 0.$$

   (a) If $x_n > 0$, show that $x_{n+1} < 0$ if and only if $x_n > K$.

   (b) Assuming that $0 < x_n < K$, show that $x_{n+1} > K$ only if $r > 4$.

   (c) What conditions on x_0 are necessary and sufficient to guarantee $x_n > 0$ for $n = 1, 2, 3, \ldots$?

5. Consider the equation

$$x_{n+1} = \frac{rx_n^2}{x_n^2 + A}$$

   (a) What are the fixed points (in terms of r and A)?

   (b) For what value(s) of r (in terms of A) are there three equilibria? For what value(s) of r are there two equilibria? For what value(s) of r is there one equilibrium?

   (c) For each of the cases described in part(b) of this question, draw a cobwebbing diagram to demonstrate the behaviour.

   (d) For particular values of r, all solutions tend to zero. What might this model be attempting to describe in terms of populations?