

Math/Stat 2300 Assignment # 2
due February 11, 2010 by 4:30pm

Please note: you must include a printout of the **entire** Maple worksheet (including comments to explain variables and expressions) for each question.

Your assignment writeup must be your own work.

1. *Logistic map.* Consider the *logistic map*:

$$x_{n+1} = rx_n(1 - x_n)$$

which, as discussed in class, is often used in population models. Here, x_n has a value between 0 and 1 representing the population and r is a parameter (> 0) which measures how quickly the population grows.

- (a) Find the equilibria for this difference equation.
- (b) Using the example worksheet **Assignment1.mw** available on the class website (along with the Maple introduction worksheet), write a procedure that accepts r as in input and returns ordered the ordered pairs $(r, x_{181}), (r, x_{182}), \dots, (r, x_{200})$. Use $x_1 = 0.1$.
- (c) Try various values of r varying between $r = 2$ and $r = 4$. Show the output for four different values in this range. Why would we choose to output the last 20 values of the sequence (instead of outputting just the last value of the sequence)?
- (d) For the value $r = 2.5$, the sequence x_n approaches an equilibrium value. What is it? Does this agree with what you found in part (a)?
- (e) Again, using the sample Maple worksheet **Assignment1.mw**, for r -values between 2.4 and 4.0 in steps of 0.05, plot the ordered pairs $(r, x_{181}), (r, x_{182}), \dots, (r, x_{200})$. The graph should represent a pitchfork. For what range of r does it appear that the sequence x_n alternates between two values?
- (f) Using Maple, graph a similar plot, this time focusing around the value $r = 3.45$. What appears to be happening there?

2. *Geometric Similarity.* (#4, p.89) Lumber cutters wish to use readily available measurements to estimate the number of board feet of lumber in a tree. Assume they measure the diameter of the tree in inches at waist height.

Develop a model that predicts board feet as a function of diameter in inches.

Test your model by showing graphically (using Maple) the correspondence between the data provided and your model.

Use the following data for your test.

x	17	19	20	23	25	28	32	38	39	41
y	19	25	32	57	71	113	123	252	259	294

The variable x is the diameter of a ponderosa pine in inches and y is the number of board feet divided by 10.

- (a) Consider the two separate assumptions that follow, allowing each to lead to a model. Analyze each model.
- (i) Assume that all trees are right-circular cylinders and are approximately the same height.
 - (ii) Assume that all trees are right-circular cylinders and that the height of the tree is proportional to the diameter.
- (b) Which model appears to be better? Why? Justify your conclusions.