Math/Stat 2300 Assignment # 2 Solutions

Question 2

(a) Consider the number of board feet as the volume.

Under the first assumption (i), that all trees are right-circular cylinders and are approximately the same height, we have that the volume of the tree is

$$V \propto Ah$$

where A is the average cross sectional area of the tree and h is the height. Since we are assuming that all trees are the same height,

$$V \propto A$$

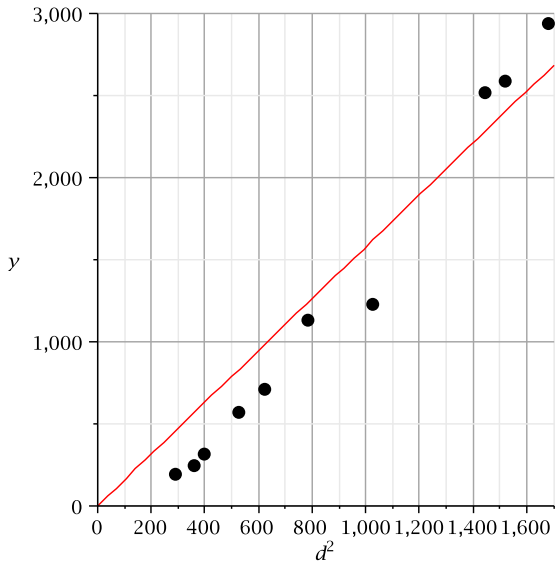
The cross sectional area is given in terms of the radius squared (πr^2). Let d is the characteristic dimension. Then we have

$$A \propto r^2 \propto d^2$$

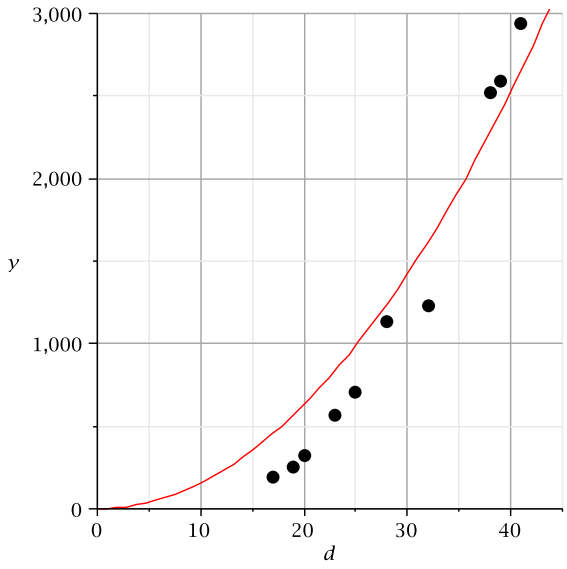
So finally, relating the volume to the characteristic dimension

$$V \propto d^2$$

Now, to determine the proportionality constant and is this assumption seems reasonable, we plot the given data y (which we are assuming is equivalent to the volume) vs. d^2 (the diameter squared)



Using Least Squares fitting: $y = 1.579186736x$



Comparing the model $y = 1.579186736d^2$ to the raw data

Under the second assumption (ii), that all trees are right-circular cylinders and that the height of the tree is proportional to the diameter, we have that the volume of the tree is again

$$V \propto Ah$$

where A is the average cross sectional area of the tree and h is the height, as above. Under this second assumption, the height is proportional to the diameter:

$$h \propto d$$

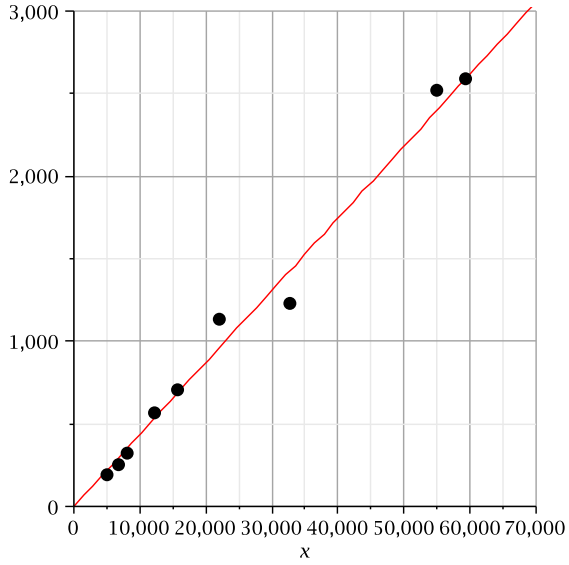
As before,

$$A \propto d^2$$

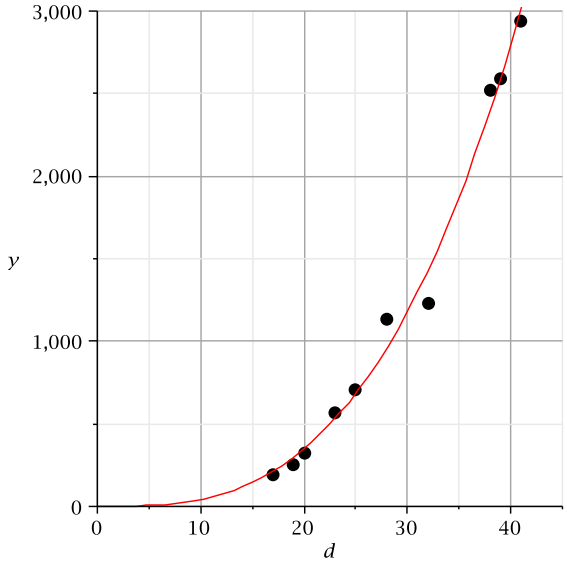
Thus,

$$V \propto d(d^2) = d^3$$

Now, to determine the proportionality constant and is this assumption seems reasonable, we plot the given data y (which we are assuming is equivalent to the volume) vs. d^3 (the diameter cubed)



Using Least Squares fitting: $y = 0.04362034724x$



Comparing the model $y = 0.04362034724d^3$ to the raw data

(b) Which model appears better?
It seems like the second model is better.