Math/Stat 2300 Midterm Make-Up Exam $\qquad$ March 18, 2010

NAME: $\qquad$ STUDENT ID:

Answer the following questions. Please include all your workings and write your solutions as clearly as possible in the space provided. Please ensure your name and student number are on all exam material. There are 10 questions in total. Please read carefully!

1. (**5 marks**) Find the solution to the difference equation

$$a_{n+1} = 0.9a_n$$

Find the equilibrium value. Determine the stability of the equilibrium value.

2. (**6 marks**) Suppose a population P grows in time according to the following differential equation

$$\frac{dP}{dt} = P - m$$

Write the equation for P as a function of t. What is one possible meaning of m? Justify.

3. (**5 marks**) The updating function of a particular difference equation is plotted below, x_n vs. x_{n+1}, along with the straight line $x_{n+1} = x_n$. Label the fixed points and use cobwebbing to determine the stability. Make a note of the stability of each equilibrium value on the graph.
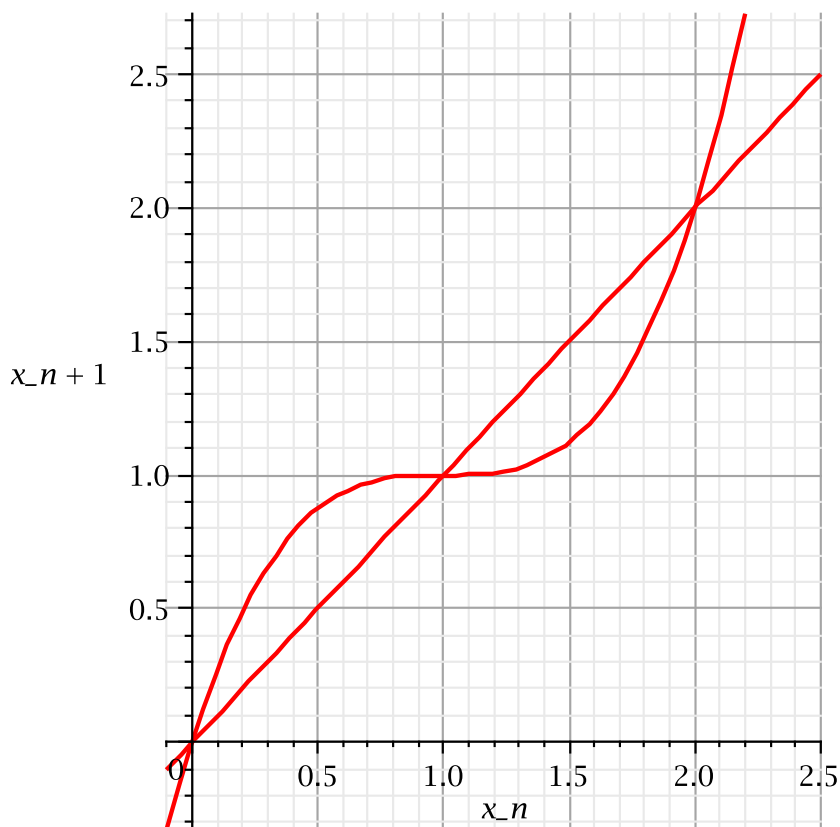
4. For each of the following models, how would we verify if a given set of data would fit the given model? How would we determine the unknown constant?

   (a) (**3 marks**) $y = Ae^{x^3}$, A is a constant

   (b) (**3 marks**) $y = 2^{bx}$, b is a constant

5. (**3 marks**) Explain the use of cubic splines for interpolation of a set of data (x_i, y_i), $i = 1, 2, \ldots m$.

6. (a) (**5 marks**) Applying Least Squares criterion to fit a straight line $y = Ax^n$, we have the following equation

$$A = \frac{\sum x_i^n y_i}{\sum x_i^{2n}}$$

Using Least Squares, fit the following data to the model $y = Ax^3$:

x	1	1.5	2	2.5
y	2	3	4	5

7. Consider the data:

x	0	2	4	6
y	0	3	5	9

(a) (**5 marks**) Using Lagrangian form of the polynomial, write the third order interpolating polynomial for the above data.

(b) (**5 marks**) Construct a divided difference table for the same data.

x	0	2	4	6
y	0	3	5	9

What is the corresponding interpolating polynomial?

8. (**5 marks**) Consider the survival of a population of whales. Assume that if the number of whales falls below a minimum survival level m, then the species will become extinct. In addition, assume that the population is limited by the carrying capacity M of the environment. Write a difference equation model to describe the whale population each year. Define all variables used. What will happen in the long run (that is, as the sequence goes to ∞)?

9. (**2 marks**) Suppose that a model has been determined for lumber cutters for the number of board feet available in a tree (that is, the volume of the tree). The diameter of the trees has been measured around the base. It has been assumed that all trees are cone-shaped (thicker on the bottom than on the top). *Recall that cones have the volume formula $V_{cone} = \frac{1}{3}\pi r^2 h$ where h is the height and r is the radius of the bottom of the cone.*
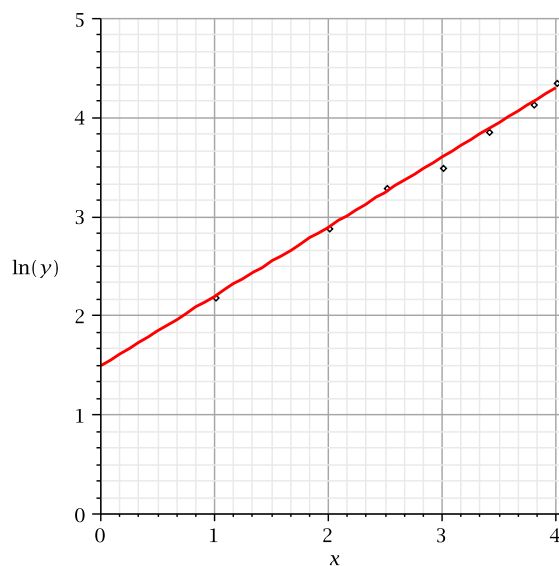
It was proposed for the model that

$$V \propto kd^2$$

where k is a constant and d is the diameter.

What is the characteristic dimension? What is one assumption that has been made?

10. (**3 marks**) In the following graph, using data (x_i, y_i) from a particular experiment, x vs $\ln(y)$ has been plotted and fitted to a straight line.

What is the model here, relating y to a function of x?