# Math/Stat 2300 Midterm Review

1. Find the solution to the difference equation

$$a_{n+1} = 0.9a_n$$

Find the equilibrium value. Determine the stability of the equilibrium value.

2. What is the equilibrium value of the difference equation

$$a_{n+1} = 0.8a_n + 100?$$

Determine the stability of the equilibrium value.

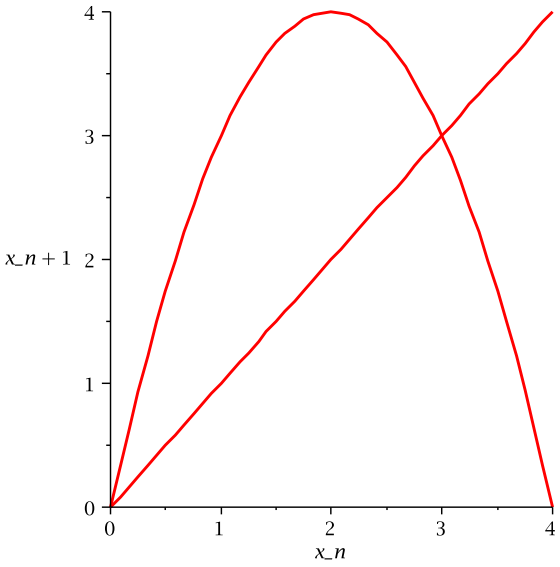
3. What is the equilibrium value of the difference equation

$$a_{n+1} = a_n - 100?$$

What is it unstable or is it stable?

4. Suppose that there are 150 university students in a residence building and some students get the flu. Write a difference equation to represent how the flu spreads through the students' residence. Define all variables. What does the proportionality constant represent?

5. A particular difference equation updating function is plotting below, $x_n$ vs. $x_{n+1}$. Label the fixed points and use cobwebbing to determine the stability. Make a note, on the graph, of the stability of the fixed points.




6. (a) Given a set of data points $(x_i, y_i)$, $i = 1, 2, \ldots, m$, fitting the data to the curve $y = f(x)$.
In terms of absolute deviations, what does the Least Squares criterion say?

(b) Applying Least Squares criterion to fit a straight line, we have the following equations

$$a = \frac{m\sum x_i y_i - \sum x_i \sum y_i}{m\sum x_i^2 - \left(\sum x_i\right)^2}$$
$$b = \frac{m\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{m\sum x_i^2 - \left(\sum x_i\right)^2}$$

Using Least Squares, fit the following data to a straight line:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 2 | 6 | 7 | 9 |

7. Consider the data:

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 1 | 2 | 7 | 22 |

(a) Using Lagrangian form of the polynomial, write the third order interpolating polynomial for the above data.

(b) Construct a divided difference table for the above data. What is the corresponding interpolating polynomial?

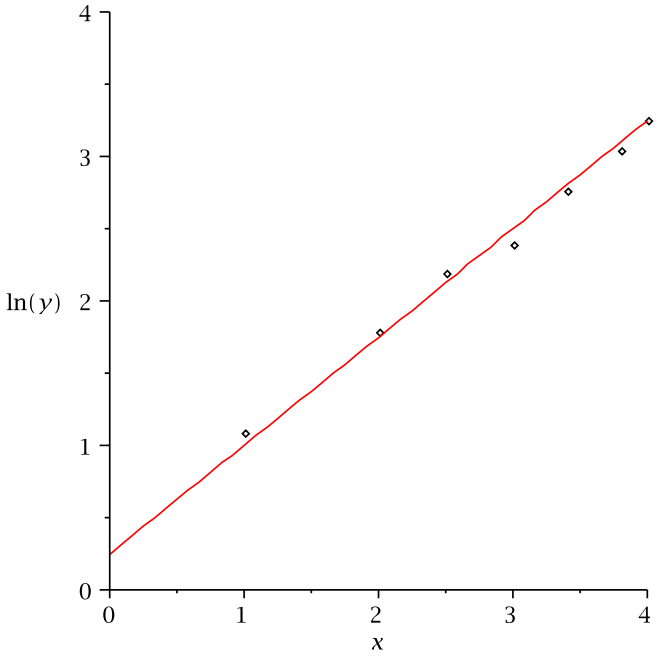
8. For each of the following models, how would we determine if a given set of data would fit the each model?

(a) $y = Ae^{bx}$

(b) $y = Ax^2$

(c) $y = Ae^{x^2}$

9. In the following graph, using data $(x_i, y_i)$ from a particular experiment, $x$ vs $\ln(y)$ has been plotted and fitted to the line $0.75x + 0.25$.




Is $x$ proportional to $\ln(y)$? What is the model, that is, what is the function relating $y$ to $x$?

10. Suppose a population $P$ grows in time according to the following differential equation

$$\frac{dP}{dt} = k(M - P), \quad P > 0$$

Write the equation for $P$ as a function of $t$. What is the meaning of the constant $k$? What is the meaning of the constant $M$?