Math 1000 Extra Problems for Final Exam Prep

Related Rates

P1. If a (spherical) snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

P2. A man 6 feet tall walks at a rate of 5 feet per second away from a street light that is 15 feet above the ground. When he is 10 feet away from the base of the light, at which rate is the tip of his shadow moving?

P3. An airplane flies at an altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rate at which the angle of elevation θ is changing when the angle is $\frac{\pi}{4}$.

P4. At noon, Ship A is 150 km west of Ship B. Ship A is travelling east at 35 km/hr and Ship B is travelling north at 25 km/hr. How fast is the distance between the ships changing at 4:00pm?

Optimization

P5. Find the dimensions of a rectangle with area 100 m^2 whose perimeter is as small as possible.

P6. Which points on the graph of $y = 4 - x^2$ are closest to the point $(0, 2)$?

P7. Find two positive numbers whose product is 192 and whose sum is a minimum.

Definite Integrals

P8. Evaluate

$$\int_1^e \frac{1}{x}\, dx.$$

P9. Evaluate

$$\int_1^2 (x^2 - 3)\, dx.$$

P10. Evaluate

$$\int_{-\pi/2}^{\pi/2} (2t + \cos(t))\, dt.$$

P11. Evaluate

$$\int_{\pi}^{\pi} \sin(x)\, dx.$$

Indefinite Integrals

P12. Integrate

$$\int (4x^3 + \frac{1}{x^2})\ dx.$$

P13. Integrate

$$\int x(x^2 + 3)\ dx.$$

P14. Integrate

$$\int (\sec^2(\theta) - \sin(\theta))\ d\theta.$$

P15. Integrate

$$\int (\cos(x) + 3^x)\ dx.$$

Integrals using Substitution

P16. Evaluate

$$\int 5e^{5x}\ dx.$$

P17. Evaluate

$$\int x(x^2 + 1)^2\ dx.$$

P18. Evaluate

$$\int \sin^2(3x)\cos(3x)\ dx.$$

P19. Evaluate

$$\int_1^5 \frac{x}{\sqrt{2x - 1}}\ dx.$$

P20. Evaluate

$$\int_1^2 e^{1-x}\ dx.$$

P21. Evaluate

$$\int_0^{\pi/2} \sin(2x)\ dx.$$