Math 1000 Final Exam Review

Note: The Final Exam is on Wednesday, August 13th, from 6pm to 9pm.
NO CALCULATORS! NO FORMULA SHEETS!

Material covered:

(From 6th Edition of Stewart) Sections 2.1, 2.2, 2.3, 2.5, 2.6, 2.7, 2.8, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 3.10, 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 4.9, 5.1, 5.2, 5.3, 5.4, 5.5

Also note that I will assume that you are familiar with precalculus material (for example, values of cosine and sine at $0, \pi/2$,etc. and equations of lines).

Topics:

- Tangent lines and secant lines
- Limits, right-hand limits and left-hand limits
- Limit laws, squeeze theorem
- Continuity (definition) at a point and on an interval, types of discontinuities
- Intermediate Value Theorem
- Limit definition of derivative, rates of change
- Rules for derivatives: polynomial functions, exponential functions, product rule, quotient rule, chain rule, trigonometric functions
- Special limits of cosine and sine: $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ and $\lim_{\theta \to 0} \frac{\cos \theta - 1}{\theta} = 1$.
- Implicit differentiation
- Derivatives of inverse trig functions
- Derivatives of logarithmic functions, logarithmic differentiation
- Exponential growth and decay
- Related rates
- Linear approximations
- Maximum and minimum values
- Mean Value Theorem and Rolle's Theorem
- Limits at infinity, horizontal asymptotes
- L'Hospital's Rule
- Curve sketching: critical numbers, intervals of increasing/decreasing, concavity, first derivative test, second derivative test, etc.
- Optimization
- Antiderivatives
- Areas under curves
- Definite integrals: Riemann sums, midpoint rule, properties of integrals
- Fundamental theorem of Calculus (parts 1 and 2)
- Indefinite integrals, net change theorem
- Substitution rule for integrals (definite and indefinite)

Sample Final

1. Consider $f(x) = \begin{cases} -2x+3 & \text{if } x < 0 \\ x^2 & x \geq 0 \end{cases}$. Find the following limits:

   (a) $\lim_{x \to 0^+} f(x)$

   (b) $\lim_{x \to 0^-} f(x)$

   (c) $\lim_{x \to 0} f(x)$

2. Find each of the following limits:

   (a) $\lim_{x \to -3} \dfrac{x^2 + x - 6}{x + 3}$

   (b) $\lim_{x \to 0} \dfrac{\sqrt{x+1} - 1}{x}$

   (c) $\lim_{x \to 0} \dfrac{\sin(4x)}{\tan(3x)}$

3. (a) Give the limit definition of the derivative.

   (b) Use the **limit definition of derivative** to find $f'(1)$ if $f(x) = \sqrt{x+3}$.

4. (a) State the Mean Value Theorem.

   (b) State Rolle's Theorem.

   (c) Let $f(x) = x^2$ on the interval $[-2, 1]$. Show that the Mean Value Theorem applies and then find all the values of c such that

   $$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

5. Differentiate the following functions:

   (a) $H(x) = xe^x + x^2 \cos(ax) + \arcsin(x)$, where a is a constant

   (b) $f(t) = \dfrac{t^2 - 1}{t^2 + 1}$

   (c) $g(x) = \arctan\left(\sqrt{x + e^x}\right)$

   (d) $f(x) = \ln(\sin(2x^2))$

6. (a) Differentiate implicitly to find the derivative of y if

   $$\sqrt{x} + \sqrt{y} = 3.$$

   (b) Find the equation of the tangent line of the function in part (a) at the point $(4, 1)$.

7. Differentiate with respect to x (use logarithmic differentiation):
   (a) $y = \dfrac{xe^x(x-1)^{3/2}}{\sqrt{x+1}}$
   (b) $y = (1+x)^{1/x}$
8. A ladder (25 feet long) is leaning against the wall of a house. The bottom of the ladder is sliding away from the wall at a rate of 2 feet/sec. How fast is the top of the ladder moving down the wall, when the bottom of the ladder is 24 feet from the wall?
9. Find the linearization $L(x)$ of $f(x) = x^4 + 3x^2$ for $a = -1$.
10. Find the following limits:
    (a) $\lim\limits_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$
    (b) $\lim\limits_{x\to\infty} x^{1/x}$
11. Given the function $f(x) = 2 + 3x - x^3$, answer the following:
    (a) What is $f'(x)$?
    (b) Find the critical numbers.
    (c) Find the intervals of increasing/decreasing.
    (d) Find the local maximum and/or the local minimum value.
    (e) Find the intervals of concavity.
    (f) What is the inflection point?
12. Given the function $f(x) = \frac{x-1}{x^2}$, answer the following:
    (a) What is the domain?
    (b) What are the intercepts?
    (c) What is the vertical asymptote? What is the horizontal asymptote?
    (d) Find the critical numbers.
    (e) Find the intervals of increasing/decreasing.
    (f) What are the local maxima or minima?
    (g) Find the intervals of concavity.
    (h) Find the inflection points.
    (i) Using parts (a)-(h), graph the function.
13. Find two numbers whose difference is 100 and whose product is a minimum.

14. (a) Find the area under the curve of $f(x) = 4 - x^2$ on the interval $[0, 2]$ using 4 approximating rectangles and right endpoints.

    (b) State the definition of the definite integral of f from a to b (as the limit of a Riemann sum).

    (c) Write $\int_0^2 (4 - x^2)\ dx$ as the limit of the Riemann sum over n intervals (Do not evaluate the limit).

15. Evaluate the following integrals.

    (a) $\displaystyle\int_1^9 \frac{x-1}{\sqrt{x}}\ dx$

    (b) $\displaystyle\int_0^{\pi/2} \cos(x)\ dx$

    (c) $\displaystyle\int sec^2(x)\ dx$

    (d) $\displaystyle\int e^x \sin(e^x)\ dx$

    (e) $\displaystyle\int_0^7 \sqrt{4+3x}\ dx$