Math 1000 Midterm Review

Note: The Midterm Exam is on Monday, July 21st, in the first part of class, i.e. from 6:00pm to 7:30pm. I will then lecture from 7:40pm to 8:45pm.

NO CALCULATORS!

Material covered:

(From 6th Edition of Stewart) Sections 2.1, 2.2, 2.3, 2.5, 2.7, 2.8, 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 3.10

Also note that I will assume that you are familiar with precalculus material (for example, values of cosine and sine at $0, \pi/2$,etc. and equations of lines).

Topics:

- tangent lines and secant lines
- limits, right-hand limits and left-hand limits
- limit laws, squeeze theorem
- continuity (definition) at a point and on an interval, types of discontinuities
- Intermediate Value Theorem
- limit definition of derivative, rates of change
- rules for derivatives: polynomial functions, exponential functions, product rule, quotient rule, chain rule, trigonometric functions
- special limits of cosine and sine: $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta\to 0} \frac{\cos\theta - 1}{\theta} = 1$.
- implicit differentiation
- derivatives of inverse trig functions
- derivatives of logarithmic functions, logarithmic differentiation
- exponential growth and decay
- related rates
- linear approximations

Sample Midterm

Note: This sample midterm may be a little longer than what you will get. Also, the solutions will be available on my website. Please try the questions first, and then look at the solutions.

1. Consider $f(t) = \begin{cases} t^2+3 & \text{if } t \leq 1 \\ t-1 & \text{if } t > 1 \end{cases}$.

   (a)Find each of the following limits:

   (i) $\lim_{t \to 1^-} f(t)$

   (ii) $\lim_{t \to 1^+} f(t)$

   (iii) $\lim_{t \to 1} f(t)$

   (b) Give the intervals over which $f(t)$ is continuous.

2. Evaluate the following limits:

   (a)
   $$\lim_{x \to 1} \frac{x^2-1}{x-1}$$

   (b)
   $$\lim_{x \to 7} \frac{\sqrt{x+2}-3}{x-7}$$

   (c)
   $$\lim_{x \to 0} 3x \cot(x)$$

3. Give the limit definition of the derivative of a function f at a number a.

4. Find the equation of the tangent line to the curve $y = \sqrt{x}$ at the point $(1, 1)$ **using the limit definition of derivative** from above.

5. Find the derivatives of the following functions:

   (a) $G(x) = x^8 + 12x^6 - 4x^4 + 10x^3 - 4x + 5$

   (b) $f(x) = 3xe^x$

   (c) $f(x) = x^3e^{3x}$

   (d) $H(t) = \cos(a^3 - t^3)$

   (e) $f(t) = \frac{2t^2-4t}{2t-6}$

6. Find the equation of the tangent line to the curve $y = \sin(x) - 2\cos(x)$ at the point $(\frac{\pi}{2}, 1)$.

7. Find dy/dx (by implicit differentiation) of
   $$x^2y^2 + xy = 4.$$

8. Find the derivative of the following:
   (a) $y = \arcsin(2x + 1)$
   (b) $f(t) = \arctan(t^2)$
   (c) $g(x) = \ln(\sqrt[5]{x})$
9. Use logarithmic differentiation to find the derivative of the function:
   (a) $y = (2x + 3)^2(5x^2 + x)^4$
   (b) $y = (\tan(x))^{1/x}$
10. A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour, the population has increased to 420. Find an expression for the number of bacteria after t hours.
11. A plane flying horizontally at an altitude of 1 mile and a speed of 500 mi/hr passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station.
12. Verify the linear approximation
$$\sqrt{x+3} \approx \frac{7}{4} + \frac{x}{4}$$
    at $a = 1$.
13. Use the Intermediate Value Theorem to show that there is a root of
$$\cos(x) = x$$
    in the interval $(0, \frac{\pi}{2})$.