Diagnostic #1 Solutions 1341A. F96

1. Find the intersection of the lines x = 1 + 2s, y = 2 - s, z = 3 - 2s and x = 3 + 5t, y = 3 - t, z = 5 - 2t.

a. (1, 0, -1)
b. (3, 4, 5)
c. (5, 7, -3)
d. (1/3) (11, -23, 17)
e. (-5, 8, 4)
f. (1/3) (-11, 13, 23)

Solve $1+2s = 3+5t$ (1)
$2-s = 3-t \Rightarrow s = -1+t$ *
$3-2s = 5-2t$ (3)

Subs. (*) in (1) gives $t = -4/3$; (*) then yields $s = -7/3$ $\Rightarrow$ $x = 1+2(-7/3) = -11/3$
$y = 2+7/3 = 13/3$
$z = 3+\frac{14}{3} = \frac{23}{3}$

& check pt with (3) ✓

2. Which of the vectors below is orthogonal to both $\overset{u}{(2, 1, -1)}$ and $\overset{v}{(-3, -2, 4)}$?

a. (2, -5, -1)
b. (1, 0, 1)
c. (-4, 0, 3)
d. (1, -2, 0)
e. (3, 0, 2)
f. None is

Any vector $\perp$ to both u and v is parallel to

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ -3 & -2 & 4 \end{vmatrix} = (2, -5, -1)$$

3. If $\mathbf{u} = (-2, 1, 1)$ and $\mathbf{v} = (1, 0, 1)$, then $\|\text{proj}_{\mathbf{v}}\mathbf{u}\|$ is:

a. 0
b. 1/2
c. $\sqrt{2}/2$
d. 1
e. $\sqrt{6}/6$
f. 1/6

$$= \frac{|u \cdot v|}{\|v\|^2}\|v\| = \frac{|u \cdot v|}{\|v\|} = \frac{|-2+1|}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

4. An equation for the intersection of the planes $5x + 7y - 4z - 8 = 0$ and $x - y + 8 = 0$ is $(x, y, z) =$

a. $(-4, 4, 0) + t\,(1, 3, 1)$
b. $(4, 0, 4) + t\,(-1, 3, 1)$
c. $(4, 4, 0) + t\,(1, 1, 8)$
d. $(4, -4, 0) + t\,(-1, 1, -3)$
e. $(0, 4, -4) + t\,(1, 3, -1)$
f. $(-4, 4, 0) + t\,(1, 1, 3)$

$$n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 5 & 7 & -4 \\ 1 & -1 & 0 \end{vmatrix} = (-4, -4, -12) \parallel \text{to } (1, 1, 3)$$

(The line of intersection of these 2 (non-$\parallel$) planes is parallel to the cross product of their normals, since this line is orthogonal to both normals)

5. If $\mathbf{u} = (3, 0, 3)$, $\mathbf{v} = (-5, 1, -8)$ and $\mathbf{w} = (0, 3, 4)$, then $\|(2\mathbf{u} + \mathbf{v}) \times \mathbf{w}\|$ is

a. $2\sqrt{5}$
b. $10\sqrt{5}$
c. $5\sqrt{5}$
d. 25
e. $5\sqrt{2}$
f. $2\sqrt{10}$

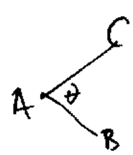
$2u+v = (6,0,6)+(-5,1,-8) = (1,1,-2)$

$(2u+v)\times w = \begin{vmatrix} i & j & k \\ 1 & 1 & -2 \\ 0 & 3 & 4 \end{vmatrix} = (10,-4,3)$

$\|(10,-4,3)\| = \sqrt{100+16+9} = \sqrt{125}$
$= 5\sqrt{5}$

6. Find the angle BAC if A = (2, 4, 1), B = (3, 0, 9) and C = (1, 4, 0).

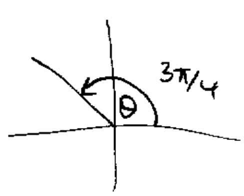
a. $4\pi/3$
b. $\pi/2$
c. $\pi/3$
d. $3\pi/4$
e. $\pi/4$
f. $\pi/6$



$\vec{AB} = (1,-4,8)$
$\vec{AC} = (-1,0,-1)$

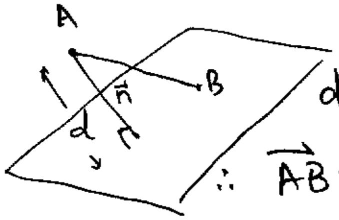
$\therefore \cos\theta = \dfrac{\vec{AB}\cdot\vec{AC}}{\|\vec{AB}\|\,\|\vec{AC}\|} = \dfrac{-9}{\sqrt{1+16+64}\,\sqrt{2}}$

$= \dfrac{-9}{\sqrt{81}\cdot\sqrt{2}} = \dfrac{-1}{\sqrt{2}}$



7. The distance from the point (4, 0, 1) to the plane $2x - y + 8z = 3$ is

a. 0
b. $17/\sqrt{69}$
c. $15/\sqrt{69}$
d. $13/\sqrt{69}$
e. $21/\sqrt{69}$
f. $19/\sqrt{69}$



$d = \|\text{proj}_{\vec{n}} \vec{AB}\|$ take $B = (0, -3, 0)$

$\therefore \vec{AB} = (-4, -3, -1)$ & $\vec{n} = (2, -1, 8)$

$$\therefore d = \frac{|\vec{AB} \cdot \vec{n}|}{\|\vec{n}\|} = \frac{|-8+3-8|}{\sqrt{4+1+64}} = \frac{13}{\sqrt{69}}$$

8. Parametric equations of the line containing (-5, 0, 1) and which is parallel to the two planes $2x - 4y + z = 0$ and $x - 3y - 2z = 1$ are:

a. $x = -5 + 5t, \quad y = -5t, \quad z = 1 - 10t$
b. $x = -5t, \quad y = 0, \quad z = t$
c. $x = 5 + 11t, \quad y = 3t, \quad z = 1 + 2t$
d. $x = -5 + 11t, \quad y = 5t, \quad z = 1 - 2t$
e. $x = 5t, \quad y = 0, \quad z = t$
f. $x = -5 + 11t, \quad y = -3t, \quad z = 1 + 2t$

The line must be orthogonal to both normals, so a direction vector is $n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 2 & -4 & 1 \\ 1 & -3 & -2 \end{vmatrix} = (11, 5, -2)$

9. The line passing through the points (2, 5, -2) and (11, -3, 22) intersects the yz-plane at the point:

a. (1/9) (0, -66, -92) $(x,y,z) = (2,5,-2) + s(9,-8,24) = (0,y,z)$
b. (0, 61, 66)
c. (0, 42, 92)
d. (0, -41, -91)
e. (0, 42, -91)
f. (1/9) (0, 61, -66)

$\therefore\ 2 + 9s = 0 \ \therefore\ s = -2/9$

so $y = 5 - 8s = 5 + \frac{16}{9} = 61/9$

$z = -2 + 24s = -2 - \frac{48}{9} = \frac{-66}{9}$

10. Which of the points (1, 0, 1) and (1, 1, 1) is closest to the point (-1, 2, -2), and what is this minimal distance?

u v w

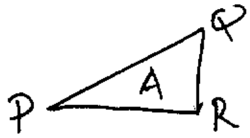
a. (1, 0, 1), 17
b. (1, 1, 1), 14
c. (1, 0, 1), $\sqrt{14}$
d. (1, 1, 1), $\sqrt{17}$
e. (1, 1, 1), $\sqrt{14}$
f. (1, 0, 1), $\sqrt{17}$

$\|u - w\| = \|(2,-2,3)\| = \sqrt{4+4+9} = \sqrt{17}$

$\|v - w\| = \|(2,-1,3)\| = \sqrt{14}$

11. Find the area of the triangle PQR where P = (1, 1, 1), Q = (2, 3, -3) and R = (2, 1, 5).

a. $3\sqrt{7}$
b. 8
c. 10
d. $4\sqrt{2}$
e. 9
f. $\sqrt{33}$



$A = \frac{1}{2}\|\vec{PR} \times \vec{PQ}\|$ $\quad \vec{PR} = (1, 0, 4)$, $\vec{PQ} = (1, 2, -4)$

$$\vec{PR} \times \vec{PQ} = \begin{vmatrix} i & j & k \\ 1 & 0 & 4 \\ 1 & 2 & -4 \end{vmatrix} = (-8, 8, 2)$$

$$\therefore A = \frac{1}{2}\sqrt{64+64+4} = \frac{\sqrt{132}}{2} = \sqrt{33}$$

12. Find an equation of the plane passing through the points (2, 3, 4) and (-1, 2, 3) and which is parallel to the vector **w** = (3, 4, 5).

$\quad u \qquad v$

a. 3x + y + z = 8
b. x + 2y - z = 9
c. 2x - 3y - 5z = 10
d. x - 2y - z + 8 = 0
e. 3x + 4y - 5z + 10 = 0
f. x - 12y + 9z = 2

$u - v = (3, 1, 1)$ is $\parallel$ to this plane

$\therefore$ a normal can be taken as

$$n = (u-v) \times w = \begin{vmatrix} i & j & k \\ 3 & 1 & 1 \\ 3 & 4 & 5 \end{vmatrix} = (1, -12, 9)$$

$\therefore$ eqn of plane is $\quad 1(x-2) - 12(y-3) + 9(z-4) = 0$

OR, (f.)

13. The polar form of $\dfrac{3\sqrt{3}\ -\ 3i}{\sqrt{2}\ +i\sqrt{2}}$ is $= \dfrac{z_1}{z_2}$; $|z_1| = \sqrt{27+9} = 6$

$\therefore z_1 = r_1 e^{i\theta_1}$ has $r_1 = 6$ & $\cos\theta_1 = \frac{\sqrt{3}}{2}$, $\sin\theta_1 = -\frac{1}{2}$

a. 3 (cos (-5π/12) + i sin (-5π/12))
b. 2 (cos (π/12) + i sin (π/12))
c. 3 (cos (5π/12) + i sin (5π/12))
d. 3 (cos (-π/12) + i sin (-π/12))
e. 6 (cos (-π/12) + i sin (-π/12))
f. 2 (cos (-5π/12) + i sin (-5π/12))

$\therefore z_1 = 6e^{-i\pi/6}$

now, $|z_2| = 2 \therefore z_2 = r_2 e^{i\theta_2}$ has

$r_2 = 2$ and $\cos\theta_2 = \sin\theta_2 = \frac{\sqrt{2}}{2}$

$\therefore z_2 = 2e^{i\pi/4}$

$\therefore \dfrac{z_1}{z_2} = \dfrac{6e^{-i\pi/6}}{2e^{i\pi/4}} = 3e^{-i(\pi/6+\pi/4)} = 3e^{-\frac{5\pi}{12}}$

14. If z = (1 - i)^2 and w = 1 - 2i , then |zw| = $|z|\,|w| = |1-i|^2 . |1-2i|$

$= (\sqrt{2})^2 . \sqrt{1+4}$

$= 2 . \sqrt{5}$

a. 40
b. $\sqrt{5}$
c. $2\sqrt{5}$
d. 20
e. 10
f. $4\sqrt{5}$

15. Evaluate Re (z) if z $=\dfrac{8+3i}{5-3i}$. $= \dfrac{8+3i}{5-3i} \cdot \dfrac{5+3i}{5+3i} = \dfrac{40-9 + i(*)}{25+9}$ real

a. 31/34
b. 31/17
c. -41/34
d. -31/34
e. 41/34
f. 41/17

$\therefore \mathrm{Re}\, z = \dfrac{31}{34}$