1341A.F96 MT # 1 27/9/96

1. Which of the following are subspaces of **F**[0,1] ?

✓ U = {f ∈ **F**[0,1] | f(0) = 0} Zero function ∈ U ✓ If f, g ∈ U, then

✗ V = {f ∈ **F**[0,1] | f(0) = 1} × 0 ∉ V

✓ W = {f ∈ **F**[0,1] | f(0) = f(1)}

$(f+g)(0) = f(0) + g(0) = 0+0 = 0$

$\therefore f+g \in U$

∴ closed under "+" ✓

If $f \in U$ & $k \in \mathbb{R}$, then

$(kf)(0) = k \cdot f(0) = k \cdot 0 = 0$

$\therefore kf \in U$ ∴ U is closed under mult^n by scalars

∴ U is a subspace of F[0,1]

a. U and W.
b. V and W.
c. None is a subspace of **F**[0,1] .
d. U and V.
e. U only .
f. W only.

The Zero function ∈ W since $0(0) = 0 = 0(1)$

If $f, g \in W$ then $(f+g)(0) = f(0) + g(0)$
$= f(1) + g(1)$
$= (f+g)(1)$

$\therefore f+g \in W$ so W is closed under "+"

If $f \in W$ and $k \in \mathbb{R}$, then $(kf)(0) = kf(0) = kf(1) = (kf)(1)$ ∴ $kf \in W$ So W is closed under mult^n by scalars.

∴ W is a Subspace of F[0,1]

2. The volume of the parallelepiped with a vertex at the origin and edges given by the vectors **i** + **j** + 2**k**, 2**j** + 5**k** and **i** + **k** is:

a. 9
b. 11
c. 7
d. 14
e. 10
f. 3

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 0 & 2 & 5 \end{vmatrix} = (1, -5, 2)$$

$$(1, -5, 2) \cdot (1, 0, 1) = 3$$

Please record your answers on the title page.

3. Which of $U = \{ (x, y, x-y) \mid x, y \in \mathbf{R} \}$, $V = \{ (x, y, x+y) \mid x, y \in \mathbf{R} \}$ and $W = \{ (x, y, xy) \mid x, y \in \mathbf{R} \}$ are subspaces of $\mathbf{R}^3$?

a. U and W only
b. V only
c. W only
d. U and V only
e. V and W only
f. U only

$U = \{ x(1,0,1) + y(0,1,-1) / x, y \in \mathbb{R} \} = \text{span}\{(1,0,1), (0,1,-1)\} \therefore$
U is a subspace of $\mathbb{R}^3$
Similarly, $V = \text{span}\{(1,0,1),(0,1,1)\}$ so V is a subspace of $\mathbb{R}^3$
However, W is not since, e.g. $(1,1,1) \in W$ but $2.(1,1,1) = (2,2,2) \notin W$.

4. Which of the following are vector spaces?
(1) $V = \{ (x, y, z) \in \mathbf{R}^3 \mid 2x - 3y + z = 0 \}$, together with the usual vector operations.
(2) $W = \{ (x, y, z) \in \mathbf{R}^3 \mid xyz = 0 \}$, together with the usual vector operations.
(3) $X = \left\{ \begin{bmatrix} 0 & a \\ b & c \end{bmatrix} \;\middle|\; a, b \text{ and } c \in \mathbf{R} \right\}$, together with the usual vector operations on real 2 by 2 matrices.

a. (3) only
b. (2) only
c. (2) and (3)
d. (1) and (2)
e. (1) only
f. (1) and (3)

(1) is a plane through the origin in $\mathbb{R}^3$ and so is a vector space

(2) is not closed under addition:
$u = (1,1,0)$ and $v = (0,0,1) \in W$ but $u+v = (1,1,1)$ is not in W. $\therefore$ W is not a vector space.

(3) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \in X$; if $u = \begin{bmatrix} 0 & a \\ b & c \end{bmatrix}$, $v = \begin{bmatrix} 0 & a' \\ b' & c' \end{bmatrix} \in X$, then $u + v = \begin{bmatrix} 0 & a+a' \\ b+b' & c+c' \end{bmatrix} \in X$;
if $u = \begin{bmatrix} 0 & a \\ b & c \end{bmatrix} \in X$ and $k \in \mathbb{R}$, then $ku = \begin{bmatrix} 0 & ka \\ kb & kc \end{bmatrix} \in X$; Thus X is a subspace of M_{22} and hence is a vector space.

5. Which one of the following is a subspace of $\mathbf{R}^4$?

a. $\{ (a, b, c, d) \in \mathbb{R}^4 \mid a = 1, b = 0 \text{ and } c + d = 1 \}$ ×
b. $\{ (a, b, c, d) \mid a > 0 \text{ and } b < 0 \}$
c. $\{ (a, b, c, d) \mid a = b = 0 \}$
d. None of these
e. $\{ (a, b, c, d) \mid a > 0 \text{ and } b > 0 \}$
f. $\{ (a, b, c, d) \mid a + b + c + d = 1 \}$

a. does not contain $(0,0,0,0)$ ×

b. - is not closed under multiplication by (-1)
- does not contain zero either

c. $= \{ (0,0,c,d) \mid c, d \in \mathbb{R} \} = \text{span}\{ (0,0,1,0), (0,0,0,1) \}$
and so is a subspace of $\mathbb{R}^4$
(we can stop now, of course...)

d. ~~use~~

e. again, e. does not contain $(0,0,0,0)$

f. does not contain $(0,0,0,0)$.

6. Let U = span{(1,2)}.

(a) Using the subspace test, carefully prove that U is a subspace of $\mathbf{R}^2$.

(b) Find an equation for U.

(c) Let W = {v ∈ $\mathbf{R}^2$ | v.u = 0 for all u ∈ U} (i.e. W is the set of vectors which are perpendicular to every vector in U) Is W a subspace of $\mathbf{R}^2$? Give reasons for your answer.

(d) Sketch U and W in the Cartesian plane.

$\frac{1}{2}$ $\frac{1}{2}$

(a) $U = \{k(1,2) \mid k \in \mathbb{R}\}$. Since $(0,0) = 0.(1,2)$, $(0,0) \in U$.

1 If $u = k(1,2)$ and $v = k'(1,2) \in U$, then $u+v = (k+k')(1,2) \in U$ so U is closed under addition.

1 If $u = k(1,2)$ and $r \in \mathbb{R}$, then $ru = (rk)(1,2) \in U$, so U is closed under mult^n by scalars. Hence, U is a subspace of $\mathbb{R}^2$.

(b) $U = \{(x, 2x) \mid x \in \mathbb{R}\} = \{(x,y) \mid y = 2x\}$. Thus

$\frac{1}{2}$ $y = 2x$ or $y - 2x = 0$ is an equation for U.

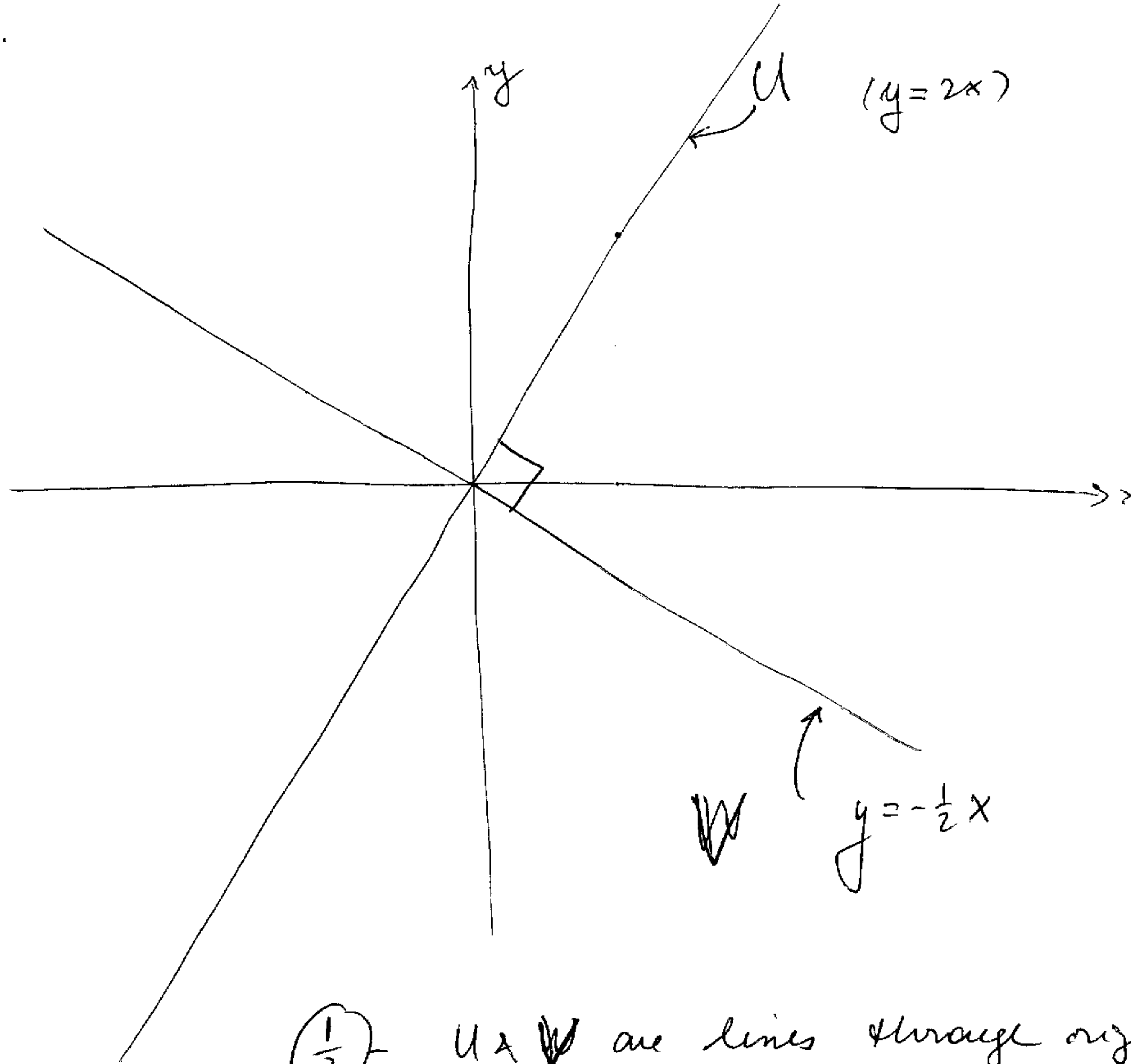
(c) $V = \{(x,y) \mid (x,y)\cdot(z,2z) = 0 \text{ for all } z \in \mathbb{R}\}$

$= \{(x,y) \mid z(x+2y) = 0 \text{ for all } z \in \mathbb{R}\}$

$= \{(x,y) \mid x+2y = 0\}$

1 Thus V is the line through the origin with ~~[illegible]~~ slope $-\frac{1}{2}$ (or: direction vector $(-2,1)$) and hence is a subspace of $\mathbb{R}^2$

(d)



(½) - U & W are lines through origin

(½) - U & V appear perpendicular

(½) - position of U & V.

7. Let $u = (1, 2, -1) \in \mathbf{R}^3$.

(a) Show that for $v = (x, y, z) \in \mathbf{R}^3$,

$$\text{proj}_u v = \tfrac{1}{6}(x + 2y - z)(1, 2, -1).$$

(b) If $v = (x, y, z)$, show that $\text{proj}_u v = (0, 0, 0)$ if and only if $x + 2y - z = 0$.

Let $W = \{v \in \mathbf{R}^3 \mid \text{proj}_u v = (0, 0, 0)\}$.

(c) Give a geometric description of W.

(d) Explain why W is a subspace of $\mathbf{R}^3$.

(a) $\text{proj}_u(x, y, z) = \underbrace{\dfrac{(x, y, z)\cdot(1, 2, -1)}{\|(1, 2, -1)\|^2}}_{1} (1, 2, -1) = \frac{1}{6}(x + 2y - z)(1, 2, -1)$

(b) $\text{proj}_u(x, y, z) = 0 \Leftrightarrow \frac{1}{6}(x + 2y - z)(1, 2, -1) = 0 \Leftrightarrow x + 2y - z = 0,$

since $(1, 2, -1) \neq 0$. 1

(c) $W = \{(x, y, z) \mid x + 2y - z = 0\}$ is the plane 1 through the origin 1 in $\mathbb{R}^3$ with normal $(1, 2, -1)$. 1

(d) W is a subspace of $\mathbb{R}^3$ because it is a plane $\frac{1}{2}$ through the origin $\frac{1}{2}$ (or a proof using the subspace test)