1341A.F96 mt2 Solutions Please return these to me. Thanks, B.

1. Given $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 4 & 1 \\ 3 & 4 \end{bmatrix}$. Compute the matrix $(AB)^{-1}$.

a. $\begin{bmatrix} 1 & -5 \\ 2 & 3 \end{bmatrix}$ b. $\begin{bmatrix} -1 & \frac{-3}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 2 \\ \frac{-5}{2} & \frac{3}{2} \end{bmatrix}$ e. $\begin{bmatrix} \frac{1}{2} & \frac{-1}{2} \\ -5 & 2 \end{bmatrix}$ f. $\begin{bmatrix} \frac{1}{2} & \frac{-5}{2} \\ 2 & 3 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 4 & 1 \\ 3 & 4 \end{bmatrix} \left(+\frac{1}{2}\right) \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix} = +\frac{1}{2} \begin{bmatrix} 1 & -5 \\ 4 & 6 \end{bmatrix}$ i.e. f.

2. Find <u>all</u> s and t so that $B^2 = 0$ if $B = \begin{bmatrix} s & t \\ 0 & s \end{bmatrix}$. $B^2 = \begin{bmatrix} s & t \\ 0 & s \end{bmatrix}\begin{bmatrix} s & t \\ 0 & s \end{bmatrix}$

$= \begin{bmatrix} s^2 & 2st \\ 0 & s^2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Leftrightarrow s = 0$

a. There are no such s and t.
b. $t = 0$ and $s \in \mathbf{R}$
c. $s = 0$ and $t = 0$
d. $s = 0$ or $t = 0$
e. $s = 0$ and $t \in \mathbf{R}$
f. $s = 1$ and $t = -1$

3. If $P = \begin{bmatrix} 1 & -4 & 1 \\ -2 & 3 & 0 \end{bmatrix}$ and $Q = \begin{bmatrix} -1 & 1 \\ 0 & 3 \\ 2 & 4 \end{bmatrix}$, which of the following is the product QP?

a. $\begin{bmatrix} 3 & -7 & 1 \\ 6 & -9 & 0 \\ 6 & -4 & -2 \end{bmatrix}$ b. $\begin{bmatrix} 3 & -4 & 1 \\ -10 & -7 & -12 \\ 3 & 0 & 3 \end{bmatrix}$

c. $\begin{bmatrix} -3 & -6 & -6 \\ 7 & 9 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ d. $\begin{bmatrix} 3 & 3 & 0 \\ -7 & -12 & -10 \\ 1 & 3 & 4 \end{bmatrix}$

e. $\begin{bmatrix} -3 & 7 & -1 \\ -6 & 9 & 0 \\ -6 & 4 & 2 \end{bmatrix}$ f. $\begin{bmatrix} 3 & -7 & 3 \\ 3 & -10 & 1 \\ 0 & -12 & 4 \end{bmatrix}$

$$\begin{bmatrix} -1 & 1 \\ 0 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -4 & 1 \\ -2 & 3 & 0 \end{bmatrix} = \begin{bmatrix} -3 & 7 & -1 \\ & * & \\ & & \end{bmatrix}$$

so the answer must be e.

4. The coefficient matrix A in a homogeneous system of 12 equations in 16 unknowns is known to have rank 6. How many free parameters are there in the solution?

a. 10
b. 6
c. 4
d. none
e. 16
f. 12

parameters = # variables − rank

= 16 − 6

= 10

[A|b] , b ≠ 0

5. For a nonhomogeneous system of 10 equations in 12 unknowns, answer the following three questions, and state which one of the suggested combinations of answers is correct.

- Can the system be inconsistent? Yes, it's possible
- Can the system have infinitely many solutions? Yes, since rank A ≤ 10 < 12, if it's consistent, it will have ∞ly many sol^ns.
- Can the system have only one solution? no. See

a. Yes, Yes, Yes
b. No, No, No
c. No, No, Yes
d. Yes, Yes, No
e. No, Yes, Yes
f. Yes, No, Yes

6. a) Find all (a,b,c) such that $\begin{bmatrix} a & b \\ a & c \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ a & c \end{bmatrix}$.

b) Let $W = \{(a,b,c) \in \mathbf{R}^3 \mid \begin{bmatrix} a & b \\ a & c \end{bmatrix}$ commutes with $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\}$

i) Explain why W is a subspace of $\mathbf{R}^3$.
ii) Give a basis of W and hence find $\dim W$.
iii) Give a geometric description of W.

a) $\begin{bmatrix} a & b \\ a & c \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ a+3c & 2a+4c \end{bmatrix}$; $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ a & c \end{bmatrix} = \begin{bmatrix} 3a & b+2c \\ 7a & 3b+4c \end{bmatrix}$

$$\therefore \quad \begin{aligned} a+3b &= 3a \\ 2a+4b &= b+2c \\ a+3c &= 7a \\ 2a+4c &= 3b+4c \end{aligned} \quad 1 \qquad \Rightarrow * \left[\begin{array}{ccc|c} -2 & 3 & 0 & 0 \\ 2 & 3 & -2 & 0 \\ -6 & 0 & 3 & 0 \\ 2 & -3 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3/2 & 0 & 0 \\ 0 & 6 & -2 & 0 \\ 0 & -9 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & -3/2 & 0 & 0 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \begin{array}{c} \\ \left[\begin{array}{ccc|c} 1 & 0 & -1/2 & 0 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \qquad \therefore \begin{aligned} a &= \tfrac{1}{2}s \\ b &= \tfrac{1}{3}s \qquad s \in \mathbb{R}. \\ c &= s \end{aligned} \quad 1$$

b) i) $W = \{(\tfrac{1}{2}s, \tfrac{1}{3}s, s) \mid s \in \mathbb{R}\} = \operatorname{span}\{(\tfrac{1}{2}, \tfrac{1}{3}, 1)\} = \operatorname{span}\{(3,2,\overset{6}{\cancel{1}})\}$ $3,2,6$

1

and so W is a subspace of $\mathbb{R}^3$. (OR any other valid reason, e.g. W is the set of sol^{n}s of the homogeneous system $*$, or W is a line through the origin in $\mathbb{R}^3$ and so is a subspace.)

ii) $\{(3,2,\overset{6}{\cancel{1}})\}$ spans W, and $(3,2,\overset{6}{\cancel{1}}) \neq 0$, so $\{(3,2,\overset{6}{\cancel{1}})\}$ is a basis of W. 1
(with justification)

Hence $\dim W = 1$. $\tfrac{1}{2}$

iii) W is the line through the origin in $\mathbb{R}^3$ with direction $(3,2,\overset{6}{\cancel{1}})$

$\tfrac{1}{2}$ $\tfrac{1}{2}$ $\tfrac{1}{2}$

7\. a) Suppose A is a 3×2 matrix and that $v = \begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. If $Av = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, explain, using block multiplication, why the columns of A must be linearly dependent. (Do not choose a numerical example for A.)

b) Suppose that B is an invertible 2×2 matrix.

i) Show that there is $\begin{bmatrix} x \\ y \end{bmatrix} \in \mathbf{R}^2$ such that $B \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

ii) Explain why $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \in \text{span}\{c_1, c_2\}$, where c_1 and c_2 are the columns of B.

(In i) and ii), do not choose a numerical example for B.)

a) Let $A = [c_1 \; c_2]$, where c_i are the columns of A.

Then $Av = 0 \Rightarrow A\begin{bmatrix} x \\ y \end{bmatrix} = 0 \Rightarrow [c_1 \; c_2]\begin{bmatrix} x \\ y \end{bmatrix} = 0 \Rightarrow xc_1 + yc_2 = 0.$

Since $\begin{bmatrix} x \\ y \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, this implies that $\{c_1, c_2\}$ is l.dependent. 1

b) i) If we let $\begin{bmatrix} x \\ y \end{bmatrix} = B^{-1}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$, then $B\begin{bmatrix} x \\ y \end{bmatrix} = B.(B^{-1}\begin{bmatrix} 1 \\ 2 \end{bmatrix})$

$= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. 1

ii) Since $B\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $[c_1 \; c_2]\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ i.e.

1 $xc_1 + yc_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, so $\Rightarrow$ $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \in \text{span}\{c_1 \; c_2\}$.

1 (i.e. understands 'span')