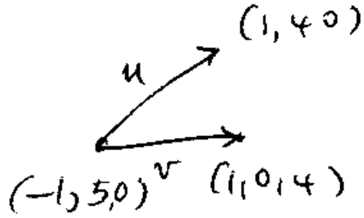
1. Find the area of the triangle with vertices (-1, 5, 0), (1, 0, 4) and (1, 4, 0).

A. 4
B. 1
C. 6
D. 3
E. 2
F. 5



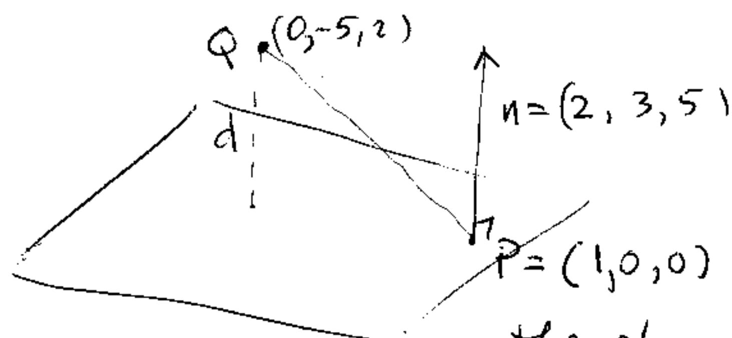
$$\text{area} = \tfrac{1}{2}\|u \times v\|$$

$$= \tfrac{1}{2}\left\| \begin{vmatrix} i & j & k \\ 2 & -5 & 4 \\ 2 & -1 & 0 \end{vmatrix} \right\|$$

$$= \tfrac{1}{2}\|(4, 8, 8)\| = 6$$

2. Find the distance of the point (0, -5, 2) from the plane 2x + 3y + 5z = 2.

A. $-\sqrt{7}/2$
B. $7/\sqrt{38}$
C. -7/6
D. $7/\sqrt{34}$
E. $-7/(4\sqrt{2})$
F. $7/\sqrt{30}$



P = (1,0,0) is on the plane

$$\therefore d = \|\operatorname{proj}_n \overrightarrow{PQ}\| = \left\| \frac{(\overrightarrow{PQ}\cdot n)}{\|n\|^2}\, n \right\| = \frac{|\overrightarrow{PQ}\cdot n|}{\|n\|}$$

$$= \frac{7}{\sqrt{38}}$$

Please record your answers on the title page.

3. Let **u** = (1, 1, 1), **v** = (0, 1, 1) and **w** = (1, 0, 1). Find the norm of $(3\mathbf{u} + \mathbf{v}) \times \mathbf{w}$.

A. $\sqrt{23}$
B. $\sqrt{53}$
C. $\sqrt{13}$
D. $\sqrt{33}$
E. $3\sqrt{7}$
F. $\sqrt{43}$

$3u + v = (3, 4, 4)$

$\therefore (3u+v) \times w = \begin{vmatrix} i & j & k \\ 3 & 4 & 4 \\ 1 & 0 & 1 \end{vmatrix}$

$= (4, +1, -4)$

$\therefore \|(3u+v) \times w\| = \|(4, 1, -4)\| = \sqrt{33}$

4. The intersection of the planes $2x - 3y + 4z = 6$ and $4x - 6y + 8z = 11$ is described by:

A. $\dfrac{x-4}{1} = \dfrac{y-5}{2} = \dfrac{z+7}{-3}$
B. The planes do not intersect
C. $\left.\begin{array}{l} x = 1 + 4t \\ y = 2 + 5t \\ z = -3 - 7t \end{array}\right\} t \in \mathbf{R}$
D. The planes are identical
E. $\dfrac{x-2}{1} = \dfrac{y+3}{2} = \dfrac{z-4}{10}$
F. $\left.\begin{array}{l} x = 1 + 2t \\ y = 2 - 3t \\ z = 10 + 4t \end{array}\right\} t \in \mathbf{R}$

$n_1 = (2, -3, 4)$ $n_2 = (4, -6, 8)$

$n_2 = 2n_1$

The normals are parallel, so the planes are identical or they don't intersect.

But since the 2nd equation isn't 2 times the first, the planes do not intersect.

5. Parametric equations for the line passing through (1, 1, -1) and which is perpendicular to the plane $2x - y + 3z = 4$, are:

A. $x = 1 + 2t,\ y = 1 - t,\ z = -1 + 3t\ ;\ t \in \mathbf{R}$
B. $x = 1 + t,\ y = 1 + t,\ z = -1 - 3t\ ;\ t \in \mathbf{R}$
C. $x = 1 - 2t,\ y = 1 + t,\ z = -1 + 3t\ ;\ t \in \mathbf{R}$
D. $x = 1 - t,\ y = 1 + t,\ z = -1 - 6t\ ;\ t \in \mathbf{R}$
E. $x = 1 - 4t,\ y = 1 - t,\ z = -1 - 3t\ ;\ t \in \mathbf{R}$
F. $x = 1 - 2t,\ y = 1 - t,\ z = -1 - 3t\ ;\ t \in \mathbf{R}$

The direction vector of the line is $(2,-1,3)$, the normal of the plane. Hence

$$v = (1,1,-1) + t(2,-1,3).$$

6. A line is parallel to the vector (-1, 2, -3) and passes through the point (0, -5, 2). Find the point where this line intersects the x–z coordinate plane.

A. (-5/2, 0, 19/2)
B. (5/2, 0, -19/2)
C. (-5/2, 0, -11/2)
D. (-5/2, 0, 11/2)
E. (5/2, 0, 19/2)
F. (5/2, 0, 11/2)

$$v = (0,-5,2) + t(-1,2,-3).$$

The x-z coordinate plane has $y = 0$ so we solve

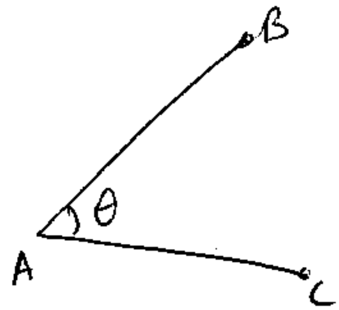
$$0 = y = -5 + t \cdot 2 \Rightarrow t = 5/2.$$

Hence the point is

$$(0,-5,2) + \tfrac{5}{2}(-1,2,-3) = \left(-\tfrac{5}{2}, 0, -11/2\right)$$

7. For the points A = (-1, 5, 0), B = (1, 0, 4) and C = (1, 4, 0), find the cosine of the angle BAC.

A. 2/5
B. 1/5
C. 6/5
D. 3/5
E. 8/5
F. 4/5



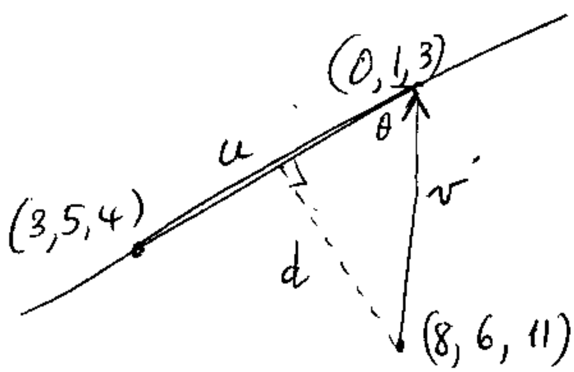
$$\cos\theta = \frac{\vec{AB}\cdot\vec{AC}}{\|\vec{AB}\|\,\|\vec{AC}\|}$$

$$= \frac{(2,-5,4)\cdot(2,-1,0)}{\sqrt{4+25+16}\cdot\sqrt{4+1}}$$

$$= \frac{9}{\sqrt{45}\cdot\sqrt{5}} = \frac{9}{3(\sqrt{5})^2} = \frac{3}{5}$$

8. Find the distance from the point (8, 6, 11) to the line containing the points (0, 1, 3) and (3, 5, 4).

A. 7
B. 3
C. 5
D. 1
E. 11
F. 9



Now $d = \|v\|\sin\theta$

and $\|u \times v\| = \|u\|\,\|v\|\sin\theta$

$\therefore d = \dfrac{\|u\times v\|}{\|u\|}$

$$\|u\times v\| = \left\| \begin{vmatrix} i & j & k \\ -3 & -4 & -1 \\ -8 & -5 & -8 \end{vmatrix} \right\| = \|(27,-16,-17)\|$$

$$\therefore d^2 = \frac{27^2+16^2+17^2}{9+16+1} = \frac{1274}{26} = \frac{637}{13} = 49.$$

Hence $d = 7$

16×16: 96, 16, 256; 17×17: 119, 17, 289; 27×27: 189, 540, 729

$637 \div 13 = 49$

$729 + 289 + 256 = 1274$

9. The point P = (2, -1, 1) lies in the plane π. If O is the origin and OP is perpendicular to π, then an equation of π is:

A. $-y - z = 0$
B. $y + 2x = 3$
C. $x - y - z = 2$
D. $x - y + 2z = 5$
E. $2x - y + z = 6$
F. $x - y - 2z = 1$

$\vec{OP} = (2,-1,1)$ is a normal

$\therefore\ 2(x-2) - 1(y+1) + (z-1) = 0$

ie $2x - y + z = 6$

10. A vector of length $\sqrt{3}$ which is orthogonal to $\underset{u}{\mathbf{3i + 4j - 5k}}$ and $\underset{v}{\mathbf{-i + 2j + k}}$ is

A. (1, 5, 7)
B. (7/5, 1/5, 1)
C. (-7/5, 7/5, -1)
D. (-1/5, 7/5, 7/5)
E. (7/5, -1/5, -5)
F. (7/5, 1/5, -1)

$u \times v$ is orthogonal to u and v, so

$$u \times v = \begin{vmatrix} i & j & k \\ 3 & 4 & -5 \\ -1 & 2 & 1 \end{vmatrix} = (14, +2, 10)$$

So we want $a.(14,2,10)$ to have length $\sqrt{3}$.

But $\|(14,2,10)\| = 2\sqrt{49+1+25} = 2\sqrt{75} = 10\sqrt{3}$

Hence we want $a = \frac{1}{10}$

11. Find the point (if one exists) where the lines $x - 1 = -3t$, $y + 2 = 4t$, $z - 3 = t$ and $x - 5 = 4t$, $y + 3 = -t$, $z - 5 = 2t$ intersect.

A. (1, -2, 3)
B. The lines do not intersect
C. (4, 5, 6)
D. (1, -2, 3)
E. (-4/7, -1, -2)
F. (0, 6, -3)

$x: 1-3t = 5+4s$

$y: -2+4t = -3-s$

$z: 3+t = 5+2s$

z: gives $t = 2+2s$. Substituting in x gives

$1-(6+6s) = 5+4s$ i.e. $-5-6s = 5+4s$ or $s=-1$, so $t=0$

Substituting these in y gives $-2+0 = -3+1$, which is consistent. Hence the pt. of intersection is $(1,-2,3)$.

12. Write $\dfrac{5+6i}{2+4i} - \dfrac{4-2i}{11i}$ in the form $a + bi$.

A. $\dfrac{31}{110} i - \dfrac{207}{55}$
B. $\dfrac{207}{110} - \dfrac{2}{55} i$
C. $-\dfrac{207}{55} + \dfrac{31}{110} i$
D. $\dfrac{207}{55} - \dfrac{31}{110} i$
E. $\dfrac{207}{55} + \dfrac{31}{110} i$
F. $\dfrac{207}{110} + \dfrac{2}{55} i$

$= \dfrac{(5+6i)(2-4i)}{4+16} - \dfrac{(4-2i)(-11i)}{121}$

$= \dfrac{(10+24)-8i}{20} + \dfrac{22+44i}{121}$

$= \dfrac{17-4i}{10} + \dfrac{2+4i}{11}$

$= \dfrac{187-44i+20+40i}{110} = \dfrac{207-4i}{110}$

$= \dfrac{207}{110} - \dfrac{2}{55} i$

13. What is the polar form of $3\sqrt{3} - 3i$? $= re^{i\theta}$

A. $6\,(\cos(\pi/3) + i \sin(\pi/3))$
B. $6\,(\cos(-\pi/6) + i \sin(-\pi/6))$
C. $36\,(\cos(-\pi/6) + i \sin(-\pi/6))$
D. $36\,(\cos(\pi/6) + i \sin(\pi/6))$
E. $6\,(\cos(\pi/6) + i \sin(\pi/6))$
F. $36\,(\cos(-\pi/3) + i \sin(-\pi/3))$

$r^2 = 9.3 + 9 = 36$

$\Rightarrow r = 6$

$$\left.\begin{aligned} \cos\theta &= \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2} \\ \sin\theta &= \frac{-3}{6} = -\frac{1}{2} \end{aligned}\right\} \Rightarrow \theta = -\pi/6$$

$$\therefore\; z = 6e^{-i\pi/6} = 6\left[\cos(-\pi/6) + i\sin(-\pi/6)\right]$$

14. Choose the correct responses to the questions (i) – (i i i) from the possibilities below. If z is a complex number,

(i) Is it possible that $\bar{z} = z$? Yes, if z is real.

(ii) Is it possible that $|\bar{z}| > |z|$? No, $|z| = |\bar{z}|$ for all $z \in \mathbb{C}$

(iii) Is it possible that $\bar{z} = 2z$? This implies that $|\bar{z}| = |2z|$

or, $|z| = 2|z|$

$\Leftrightarrow |z| = 0$

$\Leftrightarrow z = 0$.

A. No, No, Yes
B. Yes, No, Yes
C. Yes, Yes, No
D. Yes, No, No
E. Yes, Yes, Yes
F. No, Yes, No

Trying $z=0$, we find $\bar{z} = 0 = 2z$, So $z=0$ satisfies $\bar{z} = 2z$.

Yes, it is possibl

15. $(1 - i)^{13} =$

A. $64(1 - i)$

B. $64\sqrt{2}(-1 - i)$

C. $64(i - 1)$

D. $64(-1 + i)$

E. $64\sqrt{2}(1 - i)$

F. $-64(1 + i)$

$$\left[\sqrt{2}\, e^{i(-\pi/4)}\right]^{13} = \sqrt{2}^{13} \cdot e^{-\frac{i13\pi}{4}}$$

$$= 2^6 \cdot \sqrt{2}\; e^{i\frac{3\pi}{4}}$$ (add 4π to argument)

$$= 64\sqrt{2}\left(-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i\right)$$

$$= 64(-1 + i)$$