1341A Diag Solns

1. If $\mathbf{u} = (3, 1, 2)$, $\mathbf{v} = (-5, 2, -7)$ and $\mathbf{w} = (0, 3, 4)$, find $\|(2\mathbf{u} + \mathbf{v}) \times \mathbf{w}\|$.

a. $2\sqrt{322}$
b. $5\sqrt{26}$
c. 322
d. 0
e. 644
f. $\sqrt{645}$

$$(2u+v) \times w = \begin{vmatrix} i & j & k \\ 1 & 4 & -3 \\ 0 & 3 & 4 \end{vmatrix}$$

$$= (25, -4, 3)$$

$$\therefore \|(2u+v) \times w\| = \sqrt{625+16+9} = \sqrt{650} = \sqrt{25 \times 26}$$
$$= 5\sqrt{26}$$

2. The intersection of the planes $2x - 3y + 4z = 6$ and $4x - 6y + 8z = 11$ is described by:

$n_1 = (2, -3, 4)$ $n_2 = (4, -6, 8)$

a. $\left.\begin{array}{l} x = 1 + 2t \\ y = 2 - 3t \\ z = 10 + 4t \end{array}\right\} t \in \mathbf{R}$

b. $\left.\begin{array}{l} x = 1 + 4t \\ y = 2 + 5t \\ z = -3 - 7t \end{array}\right\} t \in \mathbf{R}$

c. The planes are identical

d. $\dfrac{x-4}{1} = \dfrac{y-5}{2} = \dfrac{z+7}{-3}$

e. $\dfrac{x-2}{1} = \dfrac{y+3}{2} = \dfrac{z-4}{10}$

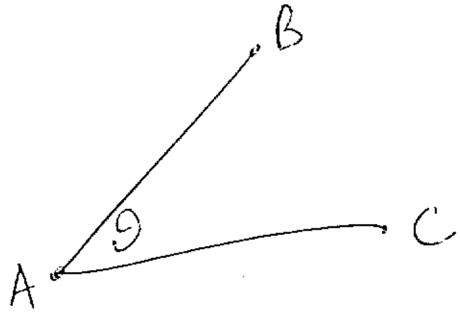
f. The planes do not intersect

$n_1 \parallel n_2$, so planes are parallel, but not identical

$(2x - 3y + 4z = 6 \Rightarrow 4x - 6y + 8z = 12)$

3. A triangle has vertices A(1, 1, 1), B(2, 3, 1) and C(1, 2, 3). Find the cosine of the interior angle at A.

a. 1
b. 3/5
c. 2/5
d. 4/5
e. 1/5
f. 0



$\vec{AB} = (1, 2, 0)$

$\vec{AC} = (0, 1, 2)$

$$\cos\theta = \frac{\vec{AB}\cdot\vec{AC}}{\|\vec{AB}\|\,\|\vec{AC}\|} = \frac{2}{\sqrt{5}\cdot\sqrt{5}} = \frac{2}{5}$$

6. An equation of the plane passing through the points P(2, 1, -1) and Q(3, 2, 1) and parallel to the y-axis is:

a. $2x - y - 5 = 0$
b. $2y - z = 5$
c. $2y - z + 5 = 0$
d. $x + y - z = 4$
e. $2x - z - 5 = 0$
f. $2x + z = 5$

A normal vector is

$$n = \vec{PQ} \times \hat{j} = \begin{vmatrix} i & j & k \\ 1 & 1 & 2 \\ 0 & 1 & 0 \end{vmatrix}$$

$$= (-2, -0, 1)$$

∴ an eqⁿ is $-2(x-2) + (z+1) = 0$

i.e. $-2x + z = -5$ or $2x - z = 5$

7. Find the point of intersection of the line passing through the points (2, 1, -3) and (5, 4, -1), and the line passing through the points (0, 2, 3) and (-1, -2, -5).

a. (-1, -2, -5)
b. (2, 1, 5)
c. (1, 2, -5)
d. (2, -1, 5)
e. (2, -1, -5)
f. (-1, -2, 5)

$$x = 2 + 3t \quad y = 1 + 3t \quad z = -3 + 2t$$

& $$x = 0 + s \quad y = 2 + 4s \quad z = 3 + 8s$$

$\therefore$ $2 + 3t = s$ so $1 + 3t = 2 + 4(2+3t) = 10 + 12t$

$\therefore$ $t = -1$. Substituting this gives $s = -1$. These values also satisfy the z- equations.

$\therefore$ $(x, y, z) = (-1, -2, -5)$.

8. If $\mathbf{u} = (3, 3, 6)$ and $\mathbf{v} = (2, -1, 1)$, the length of the projection of $\mathbf{u}$ along $\mathbf{v}$ is:

a. $(2\sqrt{6})/3$
b. 0
c. $(3\sqrt{6})/2$
d. $(2\sqrt{2})/3$
e. $(3\sqrt{2})/2$
f. $\sqrt{6}/2$

$$\| \mathrm{proj}_v (3,3,6) \| = \left\| \frac{(3,3,6)\cdot(2,-1,1)}{6} \cdot (2,-1,1) \right\|$$

$$= \left\| \frac{9}{6}(2,-1,1) \right\| = \frac{3}{2} \cdot \sqrt{6}$$

9. Parametric equations for the line passing through (1, 1, -1) and which is perpendicular to the plane $2x - y + 3z = 0$, are:

a. $x = 1 - 2t$, $y = 1 - t$, $z = -1 - 3t$; $t \in \mathbf{R}$
b. $x = 1 - 2t$, $y = 1 + t$, $z = -1 + 3t$; $t \in \mathbf{R}$
c. $x = 1 + t$, $y = 1 + t$, $z = -1 - 3t$; $t \in \mathbf{R}$
d. $x = 1 + \underline{\underline{2}}t$, $y = 1 \underline{\underline{-}}t$, $z = -1 + \underline{\underline{3}}t$; $t \in \mathbf{R}$
e. $x = 1 - t$, $y = 1 + t$, $z = -1 - 6t$; $t \in \mathbf{R}$
f. $x = 1 - 4t$, $y = 1 - t$, $z = -1 - 3t$; $t \in \mathbf{R}$

direction vector
is $(2, -1, 3)$

10. Find the x-coordinate of the point of intersection of the plane $2x + y - 3z = 2$. and the line $x = -1 + 2t$, $y = 3 + t$, $z = 2$.

a. 7/5
b. 19/5
c. -2
d. -3
e. 9/5
f. 22/5

$2(-1+2t) + (3+t) - 6 = 2$

$\Rightarrow 4t + t - 2 + 3 - 6 = 2$

i.e. $5t = 7$ $\therefore$ $t = \frac{7}{5}$

$\therefore$ $x = -1 + 2t = -1 + \frac{14}{5} = \frac{9}{5}$

11. Find all vectors in $\mathbf{R}^3$ which are orthogonal to both $\overset{u}{(-1, 1, 5)}$ and $\overset{v}{(2, 1, 2)}$.

a. (3, -12, 3) only
b. (2, -8, 2) only
c. (0, 0, 0) only
d. (-t, 0, t), where t is arbitrary
e. (t, -4t, t), where t is arbitrary
f. (t+1, -8, t+1), where t is arbitrary

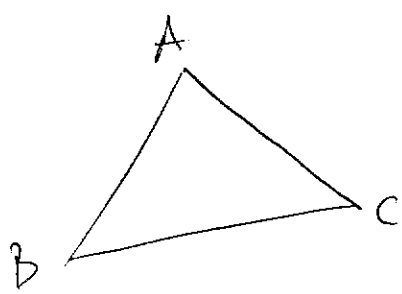
any vector $\perp$ to u and v must be parallel to $u \times v = \begin{vmatrix} i & j & k \\ -1 & 1 & 5 \\ 2 & 1 & 2 \end{vmatrix}$

i.e. $t \cdot (-3, +12, -3)$

or $t(1, -4, 1)$.

12. Find the area of the triangle whose vertices are the points $\overset{A}{(-3, 1, 2)}$, $\overset{B}{(1, -1, 0)}$ and C (1, -2, 1).

a. 24
b. 6
c. 12
d. $2\sqrt{3}$
e. $8\sqrt{3}$
f. $4\sqrt{3}$



$\overrightarrow{AB} = (4, -2, -2)$

$\overrightarrow{AC} = (4, -3, -1)$

area $= \frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \|$; $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 4 & -2 & -2 \\ 4 & -3 & -1 \end{vmatrix}$

$= (-4, -4, -4)$

$\therefore \frac{1}{2} \| (-4, -4, -4) \| = 2 \| (1, 1, 1) \| = 2\sqrt{3}$

13. if $z_1 = 3 + 2i$, $z_2 = -1 + i$, and $z_3 = 1 - 2i$, then $\dfrac{z_1 + \overline{z_2}}{z_2 z_3} = ?$

a. $5 - 5i$
b. $1/2\,(1 + i)$
c. $1/10\,(1 - 7i)$
d. $-1/10\,(1 + 7i)$
e. $1/2\,(1 - i)$
f. $5/\sqrt{10}\,(1 + i)$

$$\frac{z_1+\overline{z_2}}{z_2 z_3} = \frac{(z_1+\overline{z_2})\,\overline{z_2}\,\overline{z_3}}{|z_2|^2\,|z_3|^2}$$

$$= \frac{(2+i)(-1-i).(1+2i)}{2.5} = \frac{1}{10}(-1-3i)(1+2i)$$

$$= \frac{1}{10}(5-5i)$$

$$= \frac{1}{2}(1-i)$$

15. What is the polar form of $3\sqrt{3} - 3i$?

a. $6\,(\cos(\pi/6) + i\sin(\pi/6))$
b. $36\,(\cos(-\pi/3) + i\sin(-\pi/3))$
c. $36\,(\cos(-\pi/6) + i\sin(-\pi/6))$
d. $6\,(\cos(\pi/3) + i\sin(\pi/3))$
e. $6\,(\cos(-\pi/6) + i\sin(-\pi/6))$
f. $36\,(\cos(\pi/6) + i\sin(\pi/6))$

$$= re^{i\theta}$$

$$r = \sqrt{9.3 + 9} = 6$$

$$\cos\theta = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$$\sin\theta = \frac{-3}{6} = -\frac{1}{2}$$

$$\therefore\ \theta = -\pi/6$$

$$\therefore\ z = 6\left(\cos(-\pi/6) + i\sin(-\pi/6)\right)$$