V. Dmitri (pls check) 134 1A.F02 Diag Sol^{n}

1. An equation for the plane passing through the points (2, 1, -1) and (3, 2, 1), and parallel to the x-axis is:

A. $-3x + 7y - 2z = 3$
B. $2x - z = 5$
C. $x + y - z = 4$
D. $x - y = 1$
E. $2y - z = 3$
F. $x + y + z = 2$

$2y - z$ is the only plane, in the list which is $\parallel$ to the x-axis.

2. Find an equation of the plane which passes through the point $(1, -7, 8)$ and which is perpendicular to the line whose parametric equations are:

$$x = 2 + 2t,\ y = 7 - 4t,\ z = -3 + t;\ t \in \mathbf{R}.$$

A. $2x - 4y + z = -38$
B. $2x - 4y + z - 38 = 0$
C. $2x + 7y - 3z = -71$
D. $2x - 4y + z = -28$
E. $-4x + 2y + z + 10 = 0$
F. $-4x + 2y + z = 10$

$n = (2, -4, 1)$ $\quad v_0 = (1, -7, 8)$

$(x-1, y+7, z-8) \cdot (2, -4, 1) = 0$

$2x - 4y + z = 2 + 28 + 8 = 38$

3. Parametric equations of the line passing through $(1,\ 1,\ -1)$ and which is perpendicular to the plane $2x - y + 3z = 4$ are:

A. $x = 1 - 2t,\ y = 1 + t,\ z = -1 + 3t,\ t \in \mathbf{R}$
B. $x = 1 + 2t,\ y = 1 - t,\ z = -1 + 3t,\ t \in \mathbf{R}$
C. $x = 1 - t,\ y = 1 + t,\ z = -1 - 6t,\ t \in \mathbf{R}$
D. $x = 1 - 2t,\ y = 1 - t,\ z = -1 - 3t,\ t \in \mathbf{R}$
E. $x = 1 + t,\ y = 1 + t,\ z = -1 - 3t,\ t \in \mathbf{R}$
F. $x = 1 - 4t,\ y = 1 - t,\ z = -1 - 3t,\ t \in \mathbf{R}$

direction vector is $(2\ -1,\ 3)$ (normal to the give

$$\therefore \quad x = 1 + 2t$$
$$y = 1 - t$$
$$z = -1 + 3t$$

4. If $u = (3,\ 0,\ 3)$, $v = (-5,\ 1,\ -8)$ and $w = (0,\ 3,\ 4)$, then $\|(2u + v) \times w\|$ is:

A. $2\sqrt{5}$
B. $5\sqrt{5}$
C. $5\sqrt{2}$
D. $10\sqrt{5}$
E. 25
F. $2\sqrt{10}$

$$2u + v = (1, 1, -2)$$

$$\therefore (2u+v) \times w = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 0 & 3 & 4 \end{vmatrix}$$

$$= (10, -4, 3)$$

$$\therefore \|(2u+v) \times w\| = \sqrt{100 + 16 + 9} = \sqrt{125} = 5\sqrt{5}$$

5. Which of the vectors below is orthogonal to both (2, 1, - 1) and (-3, -2, 4)?

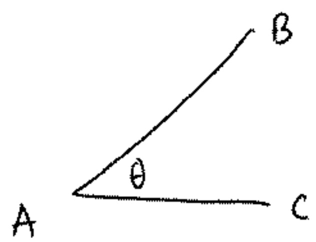
A. (1, 0, 1)
B. (2, -5, -1)
C. (1, -2, 0)
D. (-4, 0, 3)
E. (3, 0, 2)
F. None of the above

$(2,1,-1) \times (-3,-2,4)$ is $\perp$ to both:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ -3 & -2 & 4 \end{vmatrix} = (2,-5,-1)$$

6. If $A = (2, 4, 1)$, $B = (3, 0, 9)$ and $C = (1, 4, 0)$, find the angle $\angle BAC$.

A. $\pi/2$
B. $\pi/3$
C. $\pi/4$
D. $\pi/6$
E. $3\pi/4$
F. $4\pi/3$

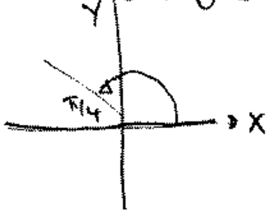


$\overrightarrow{AB} = B - A = (1,-4,8)$

$\overrightarrow{AC} = C - A = (-1,0,-1)$

$$\cos\theta = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{\|\overrightarrow{AB}\|\ \|\overrightarrow{AC}\|} = \frac{-9}{\sqrt{(1+16+64)}\sqrt{2}} = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$0 \le \theta \le \pi$



$\therefore \theta = 3\pi/4$

7. If $u = (3,\ 3,\ 3)$ and $v = (2,\ 1,\ 3)$ then $\mathrm{proj}_v u =$

$$\left(\frac{u \cdot v}{\|v\|^2}\right) v$$

A. $\frac{9}{7}(2,\ 1,\ 3)$
B. $\frac{12}{7}(2,\ 1,\ 3)$
C. $\frac{11}{7}(2,\ 1,\ 3)$
D. $\frac{9}{7}(3,\ 3,\ 3)$
E. $\frac{12}{7}(3,\ 3,\ 3)$
F. $\frac{11}{7}(3,\ 3,\ 3)$
<u>Answer:</u> A

$$= \frac{18}{(4+1+9)} \cdot (2,1,3)$$

$$= \frac{9}{7}(2,1,3)$$

8. Find the volume of the parallelepiped determined by the vectors $u = (1,\ 1,\ -1)$, $v = (2,\ 0,\ 1)$ and $w = (1,\ -1,\ 3)$.

A. -2
B. 4
C. 6
D. 8
E. 16
F. 2

vol $= |u \times v \cdot w|$;

$$u \times v = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix}$$

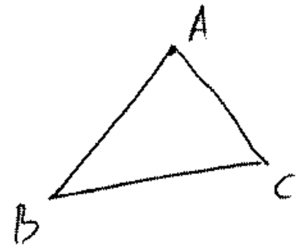
$$= (1, -3, -2)$$

$\therefore (u \times v) \cdot w = -2$ $\quad\therefore$ vol $= |-2| = 2.$

A B C

9. What is the area of the triangle with vertices (3, 0, - 2), (5, 2, -1) and (5, 9, 0)?

A. 11
B. 13
C. 15
D. 13/2
E. 15/2
F. 17/2



area $\Delta ABC = \frac{1}{2} \| \vec{BA} \times \vec{BC} \|$

$\vec{BA} = (-2, -2, -1)$; $\vec{BC} = (0, 7, 1)$

$$\vec{BA} \times \vec{BC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -2 & -1 \\ 0 & 7 & 1 \end{vmatrix} = (5, +2, -14)$$

$$\| (5, 2, -14) \|^2 = 25 + 4 + 196 = 225 = 15^2$$

$\therefore$ area $\Delta ABC = \frac{1}{2} \cdot 15.$

A B

10. Let L be the line passing through (1, 1, 0) and (2, 3, 1). The point of intersection of L with the plane $x + y - z = 1$ is:

A. (1/2, 1/2, 0)
B. (1/2, 0, −1/2)
C. (1, 0, 0)
D. (0, 1/2, −1/2)
E. (0, 1, 0)
F. (−1, 0, −1)

L has direction $\vec{AB} = (1, 2, 1)$

L has parametric eqns:

$$x = 1 + t$$
$$y = 1 + 2t \quad ; \quad t \in \mathbb{R}$$
$$z = t$$

$\therefore$ intersection is found by substitution.

$$(1+t) + (1+2t) - (t) = 1$$

$\Rightarrow 2t = -1 \quad \therefore t = -\frac{1}{2} \quad \therefore x = \frac{1}{2}$

$y = 0$ (can stop here)

$z = -\frac{1}{2}$

11. Express the following complex numbers in the form a + i b:

$$z_1 = \frac{1}{1+i} \quad = \quad \frac{1-i}{2} \quad = \quad \frac{1}{2} - \frac{i}{2}$$

$$z_2 = (2+i)(2+2i) \quad = 2 + 6i$$

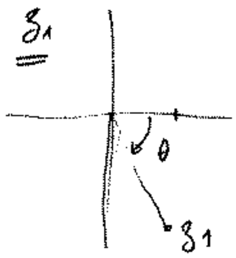
A. $z_1 = 1 - i$; $z_2 = 4 + 4i$
B. $z_1 = -1 + i$; $z_2 = 2 + 4i$
C. $z_1 = (1/2) + i(1/2)$; $z_2 = 2 - 6i$
D. $z_1 = (1/2) - i(1/2)$; $z_2 = 2 + 6i$
E. $z_1 = 2 - 1(1/4)$; $z_2 = 6 - 2i$
F. $z_1 = 1 - i$; $z_2 = 4$

12. Find the polar form of

$$\frac{1-\sqrt{3}i}{-1+i} = \frac{z_1}{z_2};$$

A. $\sqrt{2}(\cos(-5\pi/12) + i\sin(-5\pi/12))$
B. $\sqrt{2}(\cos(11\pi/12) + i\sin(11\pi/12))$
C. $\sqrt{2}(\cos(-7\pi/12) + i\sin(-7\pi/12))$
D. $\sqrt{2}(\cos(5\pi/12) + i\sin(5\pi/12))$
E. $\sqrt{2}(\cos(-\pi/12) + i\sin(-\pi/12))$
F. $\sqrt{2}(\cos(\pi/12) + i\sin(\pi/12))$

$\underline{\underline{z_1}}$

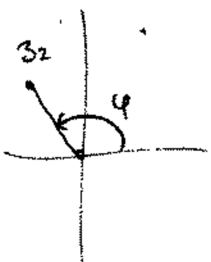


$|1-\sqrt{3}i| = \sqrt{1+3} = 2$

$\left.\begin{aligned}\cos\theta &= \tfrac{1}{2} \\ \sin\theta &= -\tfrac{\sqrt{3}}{2}\end{aligned}\right\} \therefore \theta = -\pi/3$

$\therefore z_1 = 2e^{-i\pi/3}$

$\underline{\underline{z_2}}$ $|-1+i| = \sqrt{2}$



$\left.\begin{aligned}\cos\varphi &= \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \\ \sin\varphi &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}\end{aligned}\right\} \Rightarrow \varphi = 3\pi/4 \quad \therefore z_2 = \sqrt{2}e^{i3\pi/4}$

$\therefore z_1/z_2 = \frac{2}{\sqrt{2}} e^{i(-\pi/3 - 3\pi/4)} = \sqrt{2}e^{-i\frac{13}{12}} = \sqrt{2}e^{i\frac{11}{12}}$

(add 2π to arg.)