Mat134A. F02 Final Exam Solutions

1. Which of the following are bases for $\mathbf{R}^3$?

(1) $\{(4,\ 2,\ 0),\ (1,\ 4,\ 1),\ (5,\ 6,\ 1)\}$

(2) $\{(-1,\ 2,\ 3),\ (3,\ 3,\ 2)\}$ ×

(3) $\{(-1,\ 3,\ -5),\ (1,\ -2,\ 4),\ (2,\ 0,\ 4),\ (5,\ 1,\ 9)\}$ ×

A. All three
B. (1) only
C. (2) only
D. (1) and (2)
~~E. (2) and (3)~~
F. None of them

$(5,6,1) = (4,2,0) + (1,4,1)$

$\therefore$ (1) is l.d.

2. The dimension of $S = \{A \in \mathbf{M}_{22}(\mathbf{R}) \mid \text{trace } A = 0\}$ is:

A. 0
B. 1
~~C. 2~~
D. 3
E. 4
F. S is not a vector space.

$$S = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in M_{22} \;\middle|\; a+d=0 \right\}$$

$$= \left\{ \begin{bmatrix} a & b \\ c & -a \end{bmatrix} \;\middle|\; a, b, c \in \mathbb{R} \right\}$$

But

$$\begin{bmatrix} a & b \\ c & -a \end{bmatrix} = aM_1 + bM_2 + cM_3 \quad * \qquad \therefore S = \operatorname{Span}\left\{ \underset{M_1}{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}, \underset{M_2}{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}, \underset{M_3}{\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}} \right\}$$

also * shows $\{M_1, M_2, M_3\}$ is l.i.

$\therefore \dim S = 3$

3. Consider a homogeneous system of 6 linear equations in 5 unknowns. Which of the following is true?

$$6\,\boxed{\;A\;}^{5}\,0$$

A. The system always has infinitely many solutions. ×
B. The system has only the trivial solution.
C. The system has either the trivial solution only or infinitely many solutions. ✓
D. The system can have no solution.
E. The system has between 1 and 6 solutions.
F. The system has between 0 and 5 solutions.

4. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, find the main diagonal of A^{-1}.

A. (0, 1, 0)
B. (-1, 0, -1)
C. (-1, -1, 1)
D. (0, -1, 0)
E. (-1, 0, 0)
F. (0, 0, 1)

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & 0 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{array}\right]$$

5. The dimension of the subspace of $\mathbf{R}^5$ spanned by the vectors $(1,\ 0,\ 3,\ 1,\ 1)$, $(1,\ -1,\ 7,\ -1,\ 0)$ $(2,\ 1,\ 2,\ 4,\ 3)$ and $(5,\ 1,\ 11,\ 7,\ 6)$ is:

A. 5
B. 4
C. 3
D. 2
E. 1
F. 0

$$\begin{bmatrix} 1 & 0 & 3 & 1 & 1 \\ 1 & -1 & 7 & -1 & 0 \\ 2 & 1 & 2 & 4 & 3 \\ 5 & 1 & 11 & 7 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 1 & 1 \\ 0 & -1 & 4 & -2 & -1 \\ 0 & 1 & -4 & 2 & 1 \\ 0 & 1 & -4 & 2 & 1 \end{bmatrix}$$, which

clearly has rank 2 $\therefore$ dim $= 2$

6. Which of the following are linearly independent in $\mathbf{F}(\mathbf{R}) = \{f \mid f : \mathbf{R} \to \mathbf{R}\}$.?

$S = \{\cos x, \sin x\}$
$T = \{1, \cos^2 x, \sin^2 x\}$ $\times$
$U = \{1, 2\cos^2 x, 3\sin^2 x\}$ $\times$
$V = \{2\cos x, 3\sin x\}$

$1 = \sin^2 x + \cos^2 x \quad \forall x \in \mathbb{R}$
$1 = \frac{1}{2} 2\cos^2 x + \frac{1}{3}\sin^2 x \quad$ "

A. T and V.
B. S, U and T.
C. T and U.
D. S and T.
E. S and V.
F. S, U and V.

We know S is l.i. from class, or

$a \sin x + b \cos x = 0 \quad \forall x \in \mathbb{R}$

at $x = 0$: $\quad b = 0$

$\Rightarrow$ " $x = \pi/2$: $\quad a = 0$

$\therefore a = b = 0$

$\therefore$ S is l.i.

Similarly for V

7. Compute $\left[\begin{array}{cc|cc} 1 & 0 & 0 & -3 \\ 0 & 1 & 3 & 0 \\ \hline 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]^{2002}$ $= \begin{pmatrix} I_2 & K \\ 0 & I_2 \end{pmatrix}^{2002}$

A. $\begin{bmatrix} 1 & 0 & -6006 & 0 \\ 0 & 1 & 0 & 6006 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 0 & 6006 \\ 0 & 1 & -6006 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & 0 & -3^{2002} \\ 0 & 1 & 3^{2002} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 3^{2002} \\ 0 & 1 & -3^{2002} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

F. $\begin{bmatrix} 1 & 0 & 0 & -6006 \\ 0 & 1 & 6006 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix} ; \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2k \\ 0 & 1 \end{bmatrix} = \begin{pmatrix} 1 & 3k \\ 0 & 1 \end{pmatrix}$$

$$\therefore \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}^N = \begin{pmatrix} 1 & Nk \\ 0 & 1 \end{pmatrix} \qquad \therefore \text{F}$$

8. Compute the determinant $\begin{vmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{vmatrix}$. $= \begin{vmatrix} 3 & 4 & -10 \\ 1 & 0 & 0 \\ 2 & 5 & -10 \end{vmatrix}$

A. -10
~~B. 18~~
C. 30
D. 15
E. -18
F. -36

$$= - \begin{vmatrix} 4 & -10 \\ 5 & -10 \end{vmatrix} = -(-40+50) = -10$$

9. Let A be a 4×5 matrix such that $\text{rank}(A) = 4$. If $\ker A = \{x \in \mathbf{R}^5 \mid Ax = 0\}$ and $\text{col } A = \{Ax \mid x \in \mathbf{R}^5\}$, then

col A is a s.s. of $\mathbb{R}^4$

$\dim \text{col} A = \text{rank } A = 4 \quad \therefore \text{col}(A) = \mathbb{R}^4$

A. $\text{col } A = \mathbf{R}^4$ $\dim \ker A = 4$
~~B. $\dim \text{col } A = 4, \ker A = \mathbb{R}^4$~~
C. $\text{col } A = \{0\}, \dim \ker A = 4$
D. $\dim \text{col } A = 1, \dim \ker A = 1$
~~E. $\dim \text{col } A = 1, \ker A = \mathbf{R}^5$~~
F. $\text{col } A = \mathbf{R}^4, \dim \ker A = 1$

$\dim \ker A = \#\text{cols of } A - \text{rank } A$

$= 5 - 4$

$= 1$

10. Let $A = \begin{bmatrix} 0 & 1 & 0 & -3 \\ 1 & 1 & 3 & 0 \\ 2 & 1 & 3 & 2 \\ 1 & 0 & 0 & 2 \end{bmatrix}$. If $x \in \mathbf{R}^4$, the dimension of the solution-space of $Ax = 0$ is:

A. Infinite
B. 4
C. 3
~~D. 2~~
E. 1
F. 0

$\dim \ker A = 4 - \text{rank } A;$

$$A \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 1 & 3 & -2 \\ 0 & 1 & 3 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

which has rank 3.

$\therefore \dim \ker A = 4 - 3 = 1$

11. Consider the linear system

$$\begin{array}{rcrcrcl} x & + & 3y & + & z & = & 1 \\ -x & - & 2y & + & pz & = & 1 \\ 3x & + & 7y & - & z & = & q \end{array}$$

2.5 a) If $[A\,|\,b]$ is the augmented matrix of the system above, find rank A and rank$[A\,|\,b]$ for all values of p and q.

1.5 b) Find all p and q so that this system has
i) a unique solution,
ii) infinitely many solutions, and
iii) no solutions.

2 c) In case (ii) above, give a complete geometric description of the set of solutions.

a) $\left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ -1 & -2 & p & 1 \\ 3 & 7 & -1 & q \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 1 & p+1 & 2 \\ 0 & -2 & -4 & q-3 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 1 & p+1 & 2 \\ 0 & 0 & 2p-2 & q+1 \end{array}\right]$ $\frac{1}{2}$

$\therefore \text{rank } A = \left\{\begin{array}{ll} 2 & \text{if } p=1 \\ 3 & \text{if } p \neq 1 \end{array}\right\}$ 1

$\text{rank}[A|b] = \left\{\begin{array}{ll} 3 & \text{if } p \neq 1 \\ 3 & \text{if } p=1 \text{ and } q \neq -1 \\ 2 & \text{if } p=1 \text{ and } q=-1. \end{array}\right\}$ 1

b) i) unique $\text{sol}^n \Leftrightarrow \text{rank}[A|b] = \text{rank} A = 3 \Leftrightarrow \underline{p \neq 1}$ $\frac{1}{2}$
ii) ∞ly many $\text{sol}^n \Leftrightarrow \text{rank}[A|b] = \text{rank} A < 3 \Leftrightarrow \underline{p=1,\ q=-1}$ $\frac{1}{2}$
iii) inconsistent $\Leftrightarrow \text{rank}[A|b] > \text{rank} A \Leftrightarrow \underline{p=1,\ q \neq -1.}$ $\frac{1}{2}$

c) $[A|b] \sim \left[\begin{array}{ccc|c} 1 & 3 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & -5 & -5 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\frac{1}{2}$ $\therefore$ $\begin{array}{l} x = -5+5s \\ y = 2-2s \\ z = s \end{array}$; $s \in \mathbb{R}$ $\frac{1}{2}$

This is the line in $\mathbb{R}^3$ with dirn $(5,-2,1)$, through $(-5,2,0)$.
$\frac{1}{2}$ $\frac{1}{2}$

$\frac{6}{6}$

12. Suppose $v_1 = (1,-1,0,1), v_2 = (1,-1,1,0), v_3 = (2,-2,-1,3)$, and $v_4 = (-1,1,2,-3)$ and let $W = \text{span}\{v_1, v_2, v_3, v_4\}$.

a) Find any basis of W, and so determine $\dim W$.

b) Find a basis of W which is a subset of $\{v_1, v_2, v_3, v_4\}$.

$\boxed{2}$ c) Extend your basis in (b) (if necessary) to a basis of $\mathbf{R}^4$.

a) Note: the sol^n to (b) will also be a sol^n to (a)!

Let $A = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 & -1 \\ -1 & -1 & -2 & 1 \\ 0 & 1 & -1 & 2 \\ 1 & 0 & 3 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & -1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

1 – (a)   1 (a) & (b) just^n.

$\therefore \{v_1, v_2\}$ is a basis of W $\therefore$ $\dim W = 2$. 1

b) already done   1 – (b) basis

c) Note: $\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ so

$\begin{bmatrix} v_1 \\ v_2 \\ e_2 \\ e_4 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, which

has rank 4 $\therefore$ $\{v_1, v_2, e_2, e_4\}$ is a basis of $\mathbb{R}^4$ which extends our basis in (b).

1 – any correct answer

1 – justf.

OR: Note $\begin{bmatrix} v_1 & v_2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, so $\begin{bmatrix} v_1 & v_2 & e_3 & e_4 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$,

so $\{v_1, v_2, e_3, e_4\}$ is also a basis of $\mathbb{R}^4$, extending that of (b).

13. Let $W = \{(x, y, z) \in \mathbf{R}^3 \mid x - y + z = 0\}$.

2 a) Find a basis of W and give the dimension of W.

2 b) Find an orthogonal basis of W.

2 c) Find the projection of $(1, 0, 0)$ onto W.

a) $W = \ker \begin{bmatrix} 1 & \overset{s}{-1} & \overset{t}{1} \end{bmatrix} = \{ (s-t, s, t) \mid s, t \in \mathbb{R} \}$

$= $ 1 (just that) $\operatorname{span}\{(1,1,0), (-1,0,1)\}$

Indeed, $\underline{\{ \overset{v_1}{(1,1,0)}, \overset{v_2}{(-1,0,1)} \}}$ is the basis of $\ker A$ consisting of fundamental sol'ns. $\therefore$ $\underline{\dim W = 2}$ 1

b) Set $w_1 = v_1$

$$w_2 = v_2 - \operatorname{proj}_{w_1} v_2 = (-1,0,1) - \frac{(1,1,0)\cdot(-1,0,1)}{\|(1,1,0)\|^2}(1,1,0)$$

1 - G.S.

$$= (-1,0,1) + \frac{1}{2}(1,1,0)$$

$$= \left(-\frac{1}{2}, \frac{1}{2}, 1\right)$$

$\therefore$ $\{ (1,1,0), (-\frac{1}{2}, \frac{1}{2}, 1) \}$ or $\{ \overset{w_1}{(1,1,0)}, \overset{w_2'}{(-1,1,2)} \}$ is 1

an orthogonal basis of W.

c) $$\operatorname{proj}_W (1,0,0) = \left(\frac{(1,0,0)\cdot w_1}{\|w_1\|^2} \right) w_1 + \left(\frac{(1,0,0)\cdot w_2'}{\|w_2'\|^2} \right) w_2'$$ 1

$$= \frac{1}{2}(1,1,0) + \frac{(-1)}{6}(-1,1,2)$$

$$= \left(\frac{2}{3}, \frac{1}{3}, -\frac{1}{3} \right).$$ 1

14. Let $A = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$.

1.5 a) Check that -3 and 1 are the eigenvalues of A.

1 b) Find a basis of $E_1 = \{x \in \mathbf{R}^3 \mid Ax = x\}$.

2 c) Find a basis of $E_{-3} = \{x \in \mathbf{R}^3 \mid Ax = -3x\}$.

1.5 d) Find an invertible matrix P such that $P^{-1}AP = D$ is diagonal, and give this diagonal matrix D. Show why your choice of P is invertible.

a) $|A-\lambda I| = \begin{vmatrix} 5-\lambda & 8 & 16 \\ 4 & 1-\lambda & 8 \\ -4 & -4 & -11-\lambda \end{vmatrix} = \begin{vmatrix} 5-\lambda & 8 & 16 \\ 4 & 1-\lambda & 8 \\ 0 & -3-\lambda & -3-\lambda \end{vmatrix} = -(3+\lambda) \begin{vmatrix} 5-\lambda & 8 & 16 \\ 4 & 1-\lambda & 8 \\ 0 & 1 & 1 \end{vmatrix}$

$= -(3+\lambda) \begin{vmatrix} 5-\lambda & 8 & 8 \\ 4 & 1-\lambda & 7+\lambda \\ 0 & 1 & 0 \end{vmatrix} = (\lambda+3) \begin{vmatrix} 5-\lambda & 8 \\ 4 & 7+\lambda \end{vmatrix} = (\lambda+3)\{-\lambda^2-2\lambda+35-32\} = (\lambda+3)\{-\lambda^2-2\lambda+3\}$

$1\frac{1}{2}$

$= (\lambda+3)^2(1-\lambda)$. $\therefore$ evals of A are $-3, 1$.

b) $E_1 = \ker(A-I)$; $A-I = \begin{bmatrix} 4 & 8 & 16 \\ 4 & 0 & 8 \\ -4 & -4 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 4 \\ -1 & -1 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ 0 & -1 & -1 \end{bmatrix} \sim \begin{array}{c} s \\ \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$

$\therefore E_1 = \{(-2s, -s, s) \mid s \in \mathbb{R}\}$ $\frac{1}{2}$ $\therefore \overset{\mathcal{B}_1}{\{(-2,-1,1)\}}$ is $\frac{1}{2}$ a basis of E_1 (1)

c) $E_{-3} = \ker(A+3I)$; $A+3I = \begin{bmatrix} 8 & 8 & 16 \\ 4 & 4 & 8 \\ -4 & -4 & -8 \end{bmatrix} \sim \begin{array}{c} \quad s \quad t \\ \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$ $\frac{1}{2}$ $\therefore E_{-3} = \{(-s-2t, s, t) \mid s, t \in \mathbb{R}\}$ $\frac{1}{2}$

$\therefore \overset{\mathcal{B}_{-3}}{\{(-1,1,0), (-2,0,1)\}}$ is a basis of E_{-3}.

d) Let $P = \begin{bmatrix} -2 & -1 & -2 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\frac{1}{2}$ ($\mathcal{B}_1$, $\mathcal{B}_{-3}$). Then, $\dim E_1 + \dim E_{-3} = 3 = \dim \mathbb{R}^3$, the cols of P are l.i., so P is invertible. $\frac{1}{2}$

OR: $|P| = \begin{vmatrix} -3 & 0 & -2 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} -3 & -2 \\ 1 & 1 \end{vmatrix} = -1 \neq 0$.

(assembly of $\mathcal{B}_1$ & $\mathcal{B}_3$, even if bases are incorrect, but P inv.)

Then, because the cols of P are evecs of A, $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix} = D$ is diagonal. $\frac{1}{2}$

consistent with placement of "evecs"

[7]

15. a) Let A be a real $n \times n$ matrix. Give 4 statements equivalent to "rank $A = n$", in terms of

(I) the columns of A

the cols of A are l.i. $\frac{1}{2}$

(II) the determinant of A $\neq 0$

$\frac{1}{2}$

(III) the homogeneous system $Ax = 0$ has the unique soln $x=0$ $\frac{1}{2}$

(IV) the row canonical form of A is I_n.

$\frac{1}{2}$

15b) State whether the following are true or false. If true, explain why, if false, give a numerical example to illustrate.

i) $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is diagonalizable. $\lambda = 2$ is the only eval.

(1) $A - 2I = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, which has rank 1 $\therefore$ $\dim E_1 = 2-1 = 1 < 2$,

so A is not diag'ble $\quad$ FALSE.

ii) The dimension of $W = \{v \in \mathbf{R}^5 \mid v \cdot (1,2,3,4,5) = 0\}$ is 4. ; $W = \ker [1\ 2\ 3\ 4\ 5]$;

Since rank $[1\ 2\ 3\ 4\ 5] = 1$, $\dim W = 5 - 1 = 4$. TRUE

(1) OR $W = \text{span}\{(1,2,3,4,5)\}^{\perp}$, so $\dim W = \dim \mathbb{R}^5 - \underbrace{\dim \text{span}\{(1,2,3,4,5)\}}_{1}$
$= 5 - 1$
$= 4$

iii) $\mathcal{B} = \{v_1, v_2, v_3\}$ linearly independent implies that $\mathcal{B}' = \{v_1, v_1+v_2, v_1+v_2+v_3\}$ is linearly independent.

$a v_1 + b(v_1 + v_2) + c(v_1 + v_2 + v_3) = 0 \iff$

(1) $(a+b+c)v_1 + (b+c)v_2 + c v_3 = 0 \iff$ $\begin{aligned} a + b + c &= 0 \\ b + c &= 0; \\ c &= 0 \end{aligned}$

$\mathcal{B}$ l.i.

$\Rightarrow a = b = c = 0 \therefore \mathcal{B}'$ l.i. $\Rightarrow \mathcal{B}'$ l.i. $\quad$ TRUE

iv) If A is an 11×11 matrix, and $A^t = -A$, then $\det A = 0$.

(1) $\det(A^t) = \det A$; $\det(-A) = (-1)^{11} \det A = -\det A$

$\therefore A^t = -A \Rightarrow \det A = -\det A \Rightarrow \det A = 0$.

TRUE

above (1) = $\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)$ justif.