Mat 1341F02 Mt2

1. The vectors $u_1 = (3,0,4), u_2 = (0,1,0)$, and $u_3 = (-4,0,3)$ are mutually orthogonal an each is non-zero. Find the Fourier coefficients (a_1, a_2, a_3) of $v = (0,1,1)$. That is, find $(a_1, a_2, a_3$ such that $v = a_1u_1 + a_2u_2 + a_3u_3$.

A. $(\frac{4}{25}, 1, \frac{3}{25})$
B. $(\frac{4}{5}, 1, \frac{3}{5})$
C. $(4,1,3)$
D. $(0,1,1)$
E. $(-\frac{4}{25}, 1, \frac{3}{25})$
F. $(-\frac{4}{25}, 1, -\frac{3}{25})$

$$a_1 = \frac{v \cdot u_1}{\|u_1\|^2} = \frac{4}{25}$$

(can stop here!)

$$a_2 = \frac{v \cdot u_2}{\|u_2\|^2} = 1$$

$$a_3 = \frac{v \cdot u_3}{\|u_3\|^2} = \frac{3}{25}$$

2. If G is an $n \times 2$ matrix and $H = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ then the second column of the matrix GH is

A. not defined unless $n = 2$.
B. the same as the second column of G.
C. the same as the second column of H.
D. the same as the first column of G.
E. the same as the first column of H.
F. the sum of the first and the second column of G.

$G = [g_1\ g_2]$ $\qquad GH = [g_1\ g_2]\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

g_i = i^{th} col. of G

$$= [g_1 + g_2 \quad g_2]$$

3. If $B = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, then the second row of B^{-1} is:

A. [0 1 -1]
B. [-1 1 0]
C. [0 -1 1]
D. [1 -1 0]
E. [1 0 -1]
F. None of the above

$$\left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 2 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

(Can stop here)
2nd row of RHS
doesn't change
now

4. Let A and B denote matrices, and x a column vector. If all products below are defined, which of the following statements are TRUE?

I) If either A or B is a multiple of the identity, then $AB = BA$. TRUE: See text P. 34
II) If $AB = 0$ then $A = 0$ or $B = 0$. FALSE (see your notes)
III) If A is $m \times n$ and $m < n$, the system $Ax = 0$ has infinitely many solutions. TRUE: see text P. 85

A. I and II
B. I and III
C. II and III
D. I only
E. II only
F. III only

W

5. What is the dimension of the subspace of $\mathbf{R}^4$ spanned by $(1, -1, 4, -5), (2, 1, 5, -1), (0, 1, -1$
and $(3, 4, 5, 6)$?

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5

$$W = \text{row}\begin{bmatrix} 1 & -1 & 4 & -5 \\ 2 & 1 & 5 & -1 \\ 0 & 1 & -1 & 3 \\ 3 & 4 & 5 & 6 \end{bmatrix} = \text{row}\begin{bmatrix} 1 & -1 & 4 & -5 \\ 0 & 3 & -3 & 9 \\ 0 & 1 & -1 & 3 \\ 0 & 7 & -7 & 21 \end{bmatrix}$$

$$= \text{row}\begin{bmatrix} 1 & -1 & 4 & -5 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \therefore \dim W = 2.$$

6. Let $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 5 & 10 & -4 & 19 \\ 3 & 6 & -3 & 12 \end{bmatrix}$.

1 (a) Find a basis for the row space row (A) of A.

1 (b) Find a basis for the column space col (A) of A.

1 (c) Find the dimension of $\{Ax \mid x \in \mathbf{R}^4\}$.

2 (d) Find a basis for $\ker(A) = \{x \in \mathbf{R}^4 \mid Ax = 0\}$.

1 (e) Check that $\dim\ker A + \operatorname{rank} A = 4$.

(a) $A \sim \begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A'$ $\therefore$ $\{(1,2,-1,4), (0,0,1,-1)\}$ is a basis of row(A) 1/2

1/2 — algorithm

any correct answer

(b) Since leading ones occur in cols. 1 & 3, $\{(1,5,3), (-1,-4,-3)\}$ 1 is a basis of col(A)

(c) $\{Ax \mid x \in \mathbb{R}^4\} = \operatorname{col}(A)$ 1/2 $\therefore$ its dimension $= \dim \operatorname{col}(A) = 2$ 1/2

(or any correct justification)

(d) $A' \sim \begin{array}{c} \begin{array}{cccc} & s & & t \end{array} \\ \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$ $\therefore$ $\begin{aligned} x_1 &= -2s - 3t \\ x_2 &= s \\ x_3 &= t \\ x_4 &= t \end{aligned}$ 1 * $s, t \in \mathbb{R}$ is the gen'l sol'n.

(R.C. form)

Hence $\{(-2,1,0,0), (-3,0,1,1)\}$ (i.e. the set of fundamental solutions) is a basis of ker A

1 consistent with *

(e) $\dim\ker A = 2$; 1/2 $\operatorname{rank} A = 2$ (from (a)) 1/2 $\therefore$ $\dim\ker A + \operatorname{rank} = 2 + 2 = 4$.

1.5 **7.** (a) Find all $(a, b, c) \in \mathbf{R}^3$ such that

$$(\star) \qquad \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ 0 & c \end{bmatrix}.$$

(b) Now let $W = \{(a, b, c) \in \mathbf{R}^3 \mid \star \text{ holds}\}$ be the set you found in part (a).

1 i) Show that W is a subspace of $\mathbf{R}^3$.

2 ii) Give a basis of W and hence find $\dim W$.

1.5 iii) Give a complete geometric description of W.

(a) $*$ holds iff

$$\underbrace{\overset{\text{LHS}}{\begin{bmatrix} b & -a \\ c & 0 \end{bmatrix}} = \overset{\text{RHS}}{\begin{bmatrix} 0 & -c \\ a & b \end{bmatrix}}}_{1/2} \iff \begin{aligned} b &= 0 \\ -a &= -a \\ c &= a \\ 0 &= b \end{aligned}$$

$\iff a = c$ and $b = 0$. 1

$*$ holds $\iff (a, b, c) = (a, 0, a) \qquad (a \in \mathbb{R})$.

(b) By (a) $W = \{(a, 0, a) / a \in \mathbb{R}\} = \text{span}\{(1,0,1)\} \quad (**)$

(i) W is a span $(**)$ and is therefore a subspace. 1

1/2 (just one part is necessary for 1/2)

(ii) $\{(1,0,1)\}$ spans W (see $(**)$) and $(1,0,1) \neq 0$ so $\{(1,0,1)\}$ is also l.i. 1/2 Hence $\{(1,0,1)\}$ is a basis of W. 1/2

Thus, $\dim W = 1$ 1/2

(iii) W is the line 1/2 through 0 1/2 with direction vector $(1,0,1)$. 1/2