MAT 1341 Diag. W02 Solutions.

1. An equation of the plane parallel to the vector (1, 1, -2) and which passes through the points (1, 5, 18) and (4, 2, -6) is:

A. $5x + 9y + 18z = 8$
B. $7x - 9y + 18z = 8$
C. $5x - 3y + z = 8$
D. $5x - 7y - z = 8$
E. $x + 18y + 9z = 8$
F. $9x - 6y + 5z = 8$

Solution. A normal vector of the plane is

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -2 \\ 3 & -3 & -24 \end{vmatrix} = -6(5,-3,1).$$

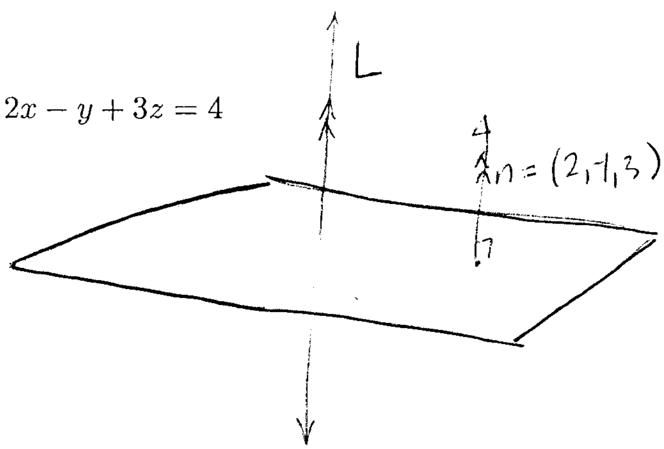
So an equation of the plane is

$$5(x-1) - 3(y-5) + (z-18) = 0$$

or $5x - 3y + z = 8$.

2. Parametric equations of the line passing through $(1, 1, -1)$ and which is perpendicular to the plane $2x - y + 3z = 4$ are:

A. $x = 1 - 2t,\ y = 1 + t,\ z = -1 + 3t,\ t \in \mathbf{R}$
B. $x = 1 + 2t,\ y = 1 - t,\ z = -1 + 3t,\ t \in \mathbf{R}$
C. $x = 1 - t,\ y = 1 + t,\ z = -1 - 6t,\ t \in \mathbf{R}$
D. $x = 1 - 2t,\ y = 1 - t,\ z = -1 - 3t,\ t \in \mathbf{R}$
E. $x = 1 + t,\ y = 1 + t,\ z = -1 - 3t,\ t \in \mathbf{R}$
F. $x = 1 - 4t,\ y = 1 - t,\ z = -1 - 3t,\ t \in \mathbf{R}$



Solution. A direction vector of the line is

$$\vec{v} = (2, -1, 3).$$

So parametric equations of the line ~~are~~:

$$x = 1 + 2t,\ y = 1 - t,\ z = -1 + 3t,\ t \in R.$$

3. If $u = (1,\ 1,\ -1)$, $v = (0,\ 2,\ -1)$, $w = (1,\ -3,\ 3)$ then the cosine of the angle between $(v \times w)$ and $(u \times v)$ is:

A. 2/21
B. -1/21
C. $\sqrt{2}/\sqrt{21}$
D. $-1/\sqrt{7}$
E. $-1/\sqrt{21}$
F. $2/\sqrt{7}$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -1 \\ 1 & -3 & 3 \end{vmatrix} = (3, -1, -2)$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & -1 \\ 0 & 2 & -1 \end{vmatrix} = (1, 1, 2)$$

$$\cos\theta = \frac{(\vec{v} \times \vec{w}) \cdot (\vec{u} \times \vec{v})}{\|\vec{v} \times \vec{w}\| \, \|\vec{u} \times \vec{v}\|}$$

$$= \frac{3 - 1 - 4}{\sqrt{9+1+4}\,\sqrt{1+1+4}} = \frac{-2}{\sqrt{14}\,\sqrt{6}} = -\frac{1}{\sqrt{21}}.$$

4. Given $\mathbf{u} = (3,\ 0,\ 3)$ and $\mathbf{v} = (-5,\ 1,\ 8)$, the orthogonal projection of $\mathbf{v}$ along $\mathbf{u}$ is:

A. (1, -1/5, -8/5)
B. (-1, 1/5, 8/5)
C. (-3/2, 0, -3/2)
D. (3/2, 0, 3/2)
E. (5, -1, -8)
F. (-5, 1, -8)

$$\mathrm{proj}_{\vec{u}}(\vec{v}) = \left(\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|^2}\right)\vec{u}$$

$$= \frac{-15+0+24}{9+0+9}(3, 0, 3)$$

$$= \frac{1}{2}(3, 0, 3) = \left(\frac{3}{2}, 0, \frac{3}{2}\right).$$

5. The volume of the parallelepiped with edges given by the vectors $u = (1,\ 1,\ 1)$, $v = (0,\ 2,\ 1)$ and $w = (1,\ 1,\ 5)$ is:

A. 4
B. $\sqrt{2}$
C. $1/\sqrt{2}$
D. $2\sqrt{2}$
E. 8
F. $8\sqrt{2}$

$$\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{vmatrix} = 8$$

The volume $= 8$.

OR:

$$u \times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix} = (-1, -1, 2)$$

$$\therefore |u \times v \cdot w| = |(-1,-1,2) \cdot (1,1,5)| = |8| = 8$$

6. Find the area of the triangle with vertices $A = (-1,\ 5,\ 0)$, $B = (1,\ 0,\ 4)$ and $C = (1,\ 4,\ 0)$.

A. 1
B. 2
C. 3
D. 4
E. 5
F. 6

$$\overrightarrow{AB} = (2, -5, 4), \quad \overrightarrow{AC} = (2, -1, 0)$$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -5 & 4 \\ 2 & -1 & 0 \end{vmatrix} = 4(1, 2, 2)$$

$$\text{Area} = \frac{1}{2} \|\overrightarrow{AB} \times \overrightarrow{AC}\| = \frac{4}{2}\sqrt{1+4+4} = 6.$$

7. Let L be the line passing through (1, 1, 0) and (2, 3, 1). The point of intersection of L with the plane $x + y - z = 1$ is:

A. (1/2, 1/2, 0)
B. (1/2, 0, −1/2)
C. (1, 0, 0)
D. (0, 1/2, −1/2)
E. (0, 1, 0)
F. (−1, 0, −1)

L: $x = 1+t,\ y = 1+2t,\ z = t,\quad t \in \mathbb{R}$.

$(1+t) + (1+2t) - t = 1 \Rightarrow t = -\frac{1}{2}$.

The intersection point is $\left(\frac{1}{2}, 0, -\frac{1}{2}\right)$.

8. Find the intersection of the lines $x = 1 + 2s,\ y = 2 - s,\ z = 3 - 2s$ and $x = 3 + 5t,\ y = 3 - t,\ z = 5 - 2t$.

A. (5, 7, -3)
B. (-5, 8, 4)
C. (3, 4, 5)
D. (1, 0, -1)
E. $\frac{1}{3}$(11, -23, 17)
F. $\frac{1}{3}$(-11, 13, 23)

$$\begin{aligned} 1+2s &= 3+5t \\ 2-s &= 3-t \\ 3-2s &= 5-2t \end{aligned} \quad \Rightarrow t = -\frac{4}{3},\ s = -\frac{7}{3}$$

The intersection point is $\left(1-\frac{14}{3}, 2+\frac{7}{3}, 3+\frac{14}{3}\right)$

$= \left(-\frac{11}{3}, \frac{13}{3}, \frac{23}{3}\right)$

$= \frac{1}{3}(-11, 13, 23)$.

9. Find the distance from the points (5, 4, 7) to the line containing the points (3, -1, 2) and (3, 1, 1).

P A B

A. 11
B. 9
C. 7
D. 5
E. 3
F. 1

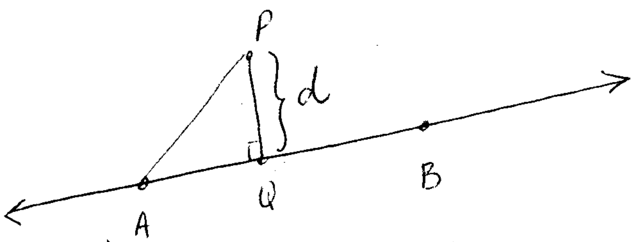
$\overrightarrow{AP}=(2,5,5),\ \overrightarrow{AB}=(0,2,-1)$

$$\text{proj}_{\overrightarrow{AB}}(\overrightarrow{AP})=\frac{0+10-5}{0+4+1}(0,2,-1)$$

$$=(0,2,-1)$$

$$d=\|\overrightarrow{AP}-\text{proj}_{\overrightarrow{AB}}(\overrightarrow{AP})\|=\|(2,5,5)-(0,2,-1)\|$$

$$=\|(2,3,6)\|=\sqrt{4+9+36}=7.$$



$\overrightarrow{AQ}=\text{proj}_{\overrightarrow{AB}}\overrightarrow{AP};\ \|\overrightarrow{PQ}\|=\|\overrightarrow{AP}-\overrightarrow{AQ}\|=d$

10. The distance from the point (4, 0, 1) to the plane $2x-y+8z=3$ is:

P

A. $13/\sqrt{69}$
B. $19/\sqrt{69}$
C. $15/\sqrt{69}$
D. 0
E. $21/\sqrt{69}$
F. $17/\sqrt{69}$

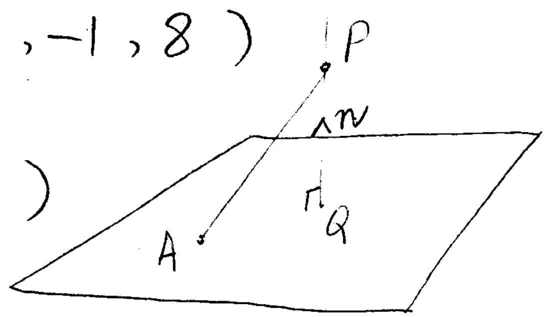
Choose a point A(0, −3, 0) on the plane.

$\overrightarrow{AP}=(4,3,1),\ \vec{n}=(2,-1,8).$

$$\text{proj}_{\vec{n}}(\overrightarrow{AP})=\frac{8-3+8}{4+1+64}(2,-1,8)$$

$$=\frac{13}{69}(2,-1,8)$$

$$d=\|\text{proj}_{\vec{n}}(\overrightarrow{AP})\|=13/\sqrt{69}.$$



$\|PQ\|=\|\text{proj}_n\overrightarrow{AP}\|$

11. Evaluate Im(z) if

$$z = \frac{1}{(1-i)(3-2i)}.$$

A. 1/2
B. -1/2
C. -1/5
D. 5/26
E. -5/26
F. 5/13

$$z = \frac{(1+i)(3+2i)}{(1+1)(9+4)} = \frac{1+5i}{26}$$

$$\operatorname{Im}(z) = \frac{5}{26}.$$

12. The polar form of

$$\frac{3\sqrt{3}-3i}{\sqrt{2}+i\sqrt{2}}$$

is:

A. $6(\cos(-\pi/12) + i\sin(-\pi/12))$
B. $3(\cos(-\pi/12) + i\sin(-\pi/12))$
C. $3(\cos(5\pi/12) + i\sin(5\pi/12))$
D. $3(\cos(-5\pi/12) + i\sin(-5\pi/12))$
E. $2(\cos(-5\pi/12) + i\sin(-5\pi/12))$
F. $2(\cos(\pi/12) + i\sin(\pi/12))$

$$3\sqrt{3} - 3i = 6e^{\frac{11\pi}{6}i},$$

$$\sqrt{2} + \sqrt{2}i = 2e^{\frac{\pi}{4}i},$$

$$\frac{3\sqrt{3}-3i}{\sqrt{2}+i\sqrt{2}} = 3e^{\left(\frac{11\pi}{6} - \frac{\pi}{4}\right)i} = 3e^{\frac{19}{12}\pi i} = 3e^{\left(\frac{19}{12}\pi - 2\pi\right)i}$$

$$= 3e^{-\frac{5\pi}{12}i} = 3\left(\cos\left(-\frac{5\pi}{12}\right) + i\sin\left(-\frac{5\pi}{12}\right)\right).$$