MAT 3321, COMPLEX ANALYSIS AND INTEGRAL TRANSFORMS, WINTER 2005

Answers to Homework 7
14.1 #14; 14.2 #2,4

Problem 14.1 #14 To show that the series $\sum_{n=0}^{\infty} \frac{i^n}{n^2+i}$ is convergent, we use the comparison test. As discussed in class, the series $\sum_{n=0}^{\infty} \frac{1}{n^2}$ converges. We also have, for all n:

$$\left|\frac{i^n}{n^2+i}\right| = \frac{1}{\sqrt{n^4+1^2}} \leqslant \frac{1}{\sqrt{n^4}} = \frac{1}{n^2}$$

Therefore, by the comparison test, the first series converges too.

Problem 14.2 #2 The power series $\sum_{n=0}^{\infty} \frac{2^{20n}}{n!}(z-3)^n$. is in powers of $z-3$, therefore the center is at $z_0 = 3$. To determine the radius of convergence, we use the Cauchy-Hadamard formula:

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{\frac{2^{20n}}{n!}}{\frac{2^{20(n+1)}}{(n+1)!}}\right| = \lim_{n\to\infty}\left|\frac{2^{20n}}{n!}\frac{(n+1)!}{2^{20(n+1)}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{2^{20n}}{n!}\frac{(n+1)n!}{2^{20n}2^{20}}\right| = \lim_{n\to\infty}\left|\frac{(n+1)}{2^{20}}\right| = \infty$$

Therefore, the series converges for all z.

Problem 14.2 #4 The power series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}\left(\frac{z}{4}\right)^{n+1} = \sum_{n=1}^{\infty} \frac{1}{n(n+1)}\frac{z^{n+1}}{4^{n+1}}$ is in powers of z, therefore the center is at $z_0 = 0$. We find the radius of convergence using the Cauchy-Hadamard formula.

$$R = \lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| = \lim_{n\to\infty}\left|\frac{\frac{1}{n(n+1)4^{n+1}}}{\frac{1}{(n+1)(n+2)4^{n+2}}}\right|$$

$$= \lim_{n\to\infty}\left|\frac{4^{n+2}}{n(n+1)}\frac{(n+1)(n+2)}{4^{n+1}}\right| = 4.$$