MAT 3361, INTRODUCTION TO MATHEMATICAL LOGIC, Fall 2004

Answers to Problem Set 2:
Handout 1, 11, 21, 31, 41, 51 (using Prawitz style natural deduction),
12, 22, 32, 42, 52 (using analytic tableaux)

Peter Selinger

Problem 11

$(a)\quad \dfrac{A \wedge (A \vee B)}{A}\,(\wedge E) \qquad\qquad (b)\quad \dfrac{A \qquad \dfrac{A}{A \vee B}\,(\vee I)}{A \wedge (A \vee B)}\,(\wedge I)$

Problem 21

$$\dfrac{\dfrac{\dfrac{[A \wedge B]_1}{B}\,(\wedge E) \qquad \dfrac{A \wedge \neg B}{\neg B}\,(\wedge E)}{\bot}\,(\neg E)}{\neg(A \wedge B)}\,(\neg I_1)$$

Problem 31

$$(a)\quad \dfrac{\dfrac{\dfrac{\dfrac{[A]_1}{A \vee B}\,(\vee I) \qquad A \vee B \to C}{C}\,(\to E)}{A \to C}\,(\to I_1) \qquad \dfrac{\dfrac{\dfrac{[B]_2}{A \vee B}\,(\vee I) \qquad A \vee B \to C}{C}\,(\to E)}{B \to C}\,(\to I_2)}{(A \to C) \wedge (B \to C)}\,(\wedge I)$$

$$(b)\quad \dfrac{\dfrac{[A \vee B]_1 \qquad \dfrac{[A]_2 \qquad \dfrac{(A \to C) \wedge (B \to C)}{A \to C}\,(\wedge E)}{C}\,(\to E) \qquad \dfrac{[B]_2 \qquad \dfrac{(A \to C) \wedge (B \to C)}{B \to C}\,(\wedge E)}{C}\,(\to E)}{C}\,(\vee E_{2,3})}{(A \vee B) \to C}\,(\to I_1)$$

Problem 41

$$\dfrac{\dfrac{[\neg(A \vee \neg A)]_1 \qquad \dfrac{\dfrac{\dfrac{[\neg(A \vee \neg A)]_1 \qquad \dfrac{[A]_2}{A \vee \neg A}\,(\vee I)}{\bot}\,(\vee E)}{\neg A}\,(\neg I_2)}{A \vee \neg A}\,(\vee I)}{\bot}\,(\neg E)}{A \vee \neg A}\,(RAA_1)$$

Problem 51

$$\dfrac{\dfrac{[\neg((A \to B) \vee (B \to A))]_1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{[\neg((A \to B) \vee (B \to A))]_1 \qquad \dfrac{\dfrac{[A]_2}{B \to A}\,(\to I_3)}{(A \to B) \vee (B \to A)}\,(\vee E)}{\bot}\,(\neg E)}{B}\,(\bot E)}{A \to B}\,(\to I_2)}{(A \to B) \vee (B \to A)}\,(\vee E)}{\bot}\,(\neg E)}{(A \to B) \vee (B \to A)}\,(RAA_1)$$

Problem 12 (a) $A \vee (A \wedge B) \vdash A$

$$\begin{array}{ll}
\multicolumn{2}{l}{FA} \\
\multicolumn{2}{l}{T(A \vee (A \wedge B))} \\
TA & T(A \wedge B) \\
 & TA \\
\times & TB \\
 & \\
 & \times
\end{array}$$

(b) $A \vdash A \vee (A \wedge B)$

$$\begin{array}{l}
TA \\
F(A \vee (A \wedge B)) \\
FA \\
\\
\times
\end{array}$$

Problem 22

$F(B \to (\neg(A \to (\neg B))))$
$T((\neg A) \to (\neg B))$
TB
$F(\neg(A \to (\neg B)))$
$T(A \to (\neg B))$

- $F(\neg A)$, TA
  - FA — $\times$
  - $T(\neg B)$, FB — $\times$
- $T(\neg B)$, FB — $\times$

Problem 32

$F(A \to (B \vee C))$
$T((A \to B) \vee (A \to C))$
TA
$F(B \vee C)$
FB
FC

- $T(A \to B)$
  - FA — $\times$
  - TB — $\times$
- $T(A \to C)$
  - FA — $\times$
  - TC — $\times$

Problem 42

$T((A \to B) \vee C)$
$T(A \to \neg C)$
$F((B \to C) \to \neg A)$
$T(B \to C)$
$F(\neg A)$
TA

- $T(A \to B)$
  - FA — $\times$
  - TB
    - FA — $\times$
    - $T(\neg C)$, FC
      - FB — $\times$
      - TC — $\times$
- TC
  - FA — $\times$
  - $T(\neg C)$, FC — $\times$

Problem 52

$T(A \to (B \vee C))$
$F((A \to B) \vee (A \to C))$
$F(A \to B)$
$F(A \to C)$
TA
FB
TA
FC

- FA — $\times$
- $T(B \vee C)$
  - TB — $\times$
  - TC — $\times$