2D Linear flows

■ Initialize

■ The various flows

■ Saddle: $\lambda_1 < 0 < \lambda_2$

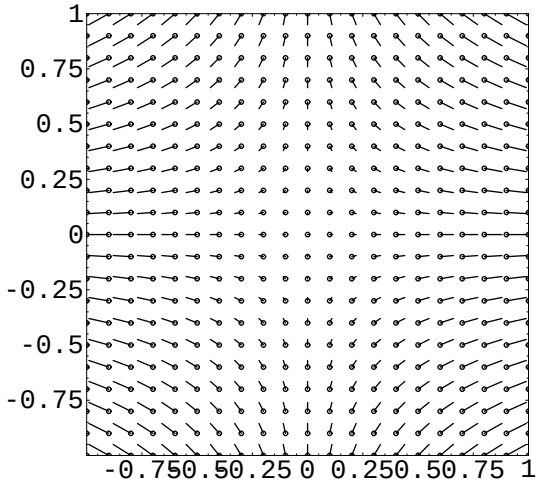
Here the matrix is in JCF

```
(A = {{2, 0}, {0, -1}}) // MatrixForm
MatrixExp[A t] // MatrixForm
```

$$\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$$

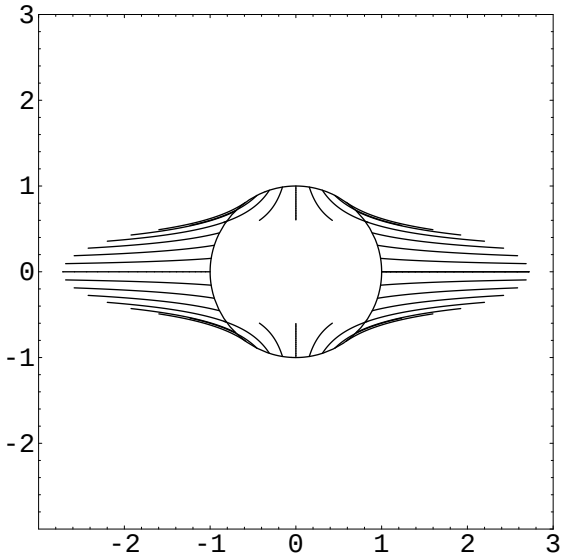
$$\begin{pmatrix} e^{2t} & 0 \\ 0 & e^{-t} \end{pmatrix}$$

```
ShowVF[A]
```



- Graphics -

```
ShowFlow[A, 0.5]
```



- Graphics -

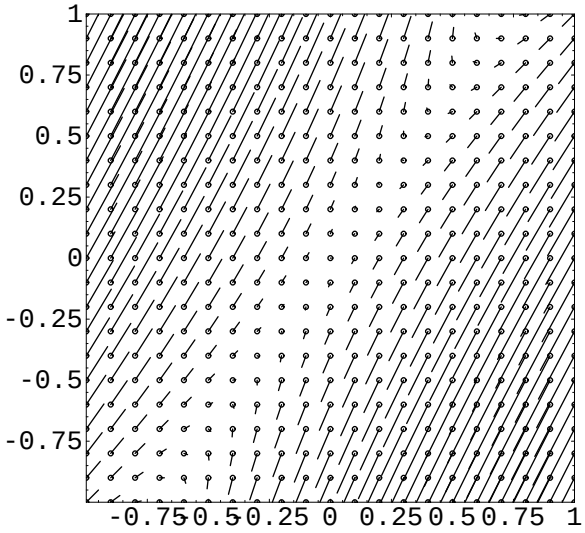
Same flow, but relative to non−standard basis

```
(J = {{2, 0}, {0, -1}});
P = {{1, 1}, {3, 1}};
A = P . J . Inverse[P];
A // MatrixForm
MatrixExp[A t] // MatrixForm
```

$$\begin{pmatrix} -\frac{5}{2} & \frac{3}{2} \\ -\frac{9}{2} & \frac{7}{2} \end{pmatrix}$$

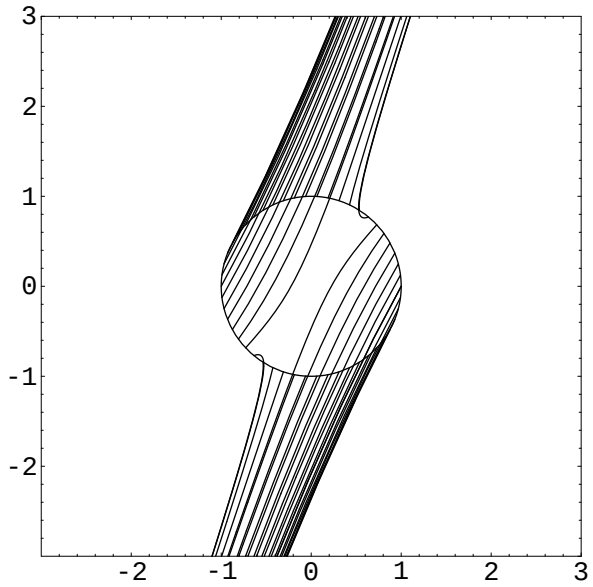
$$\begin{pmatrix} -\frac{1}{2} e^{-t} \left(-3 + e^{3t}\right) & \frac{1}{2} e^{-t} \left(-1 + e^{3t}\right) \\ -\frac{3}{2} e^{-t} \left(-1 + e^{3t}\right) & \frac{1}{2} e^{-t} \left(-1 + 3 e^{3t}\right) \end{pmatrix}$$

```
ShowVF[A]
```



- Graphics -

```
ShowFlow[A, 3, 50]
```



- Graphics -

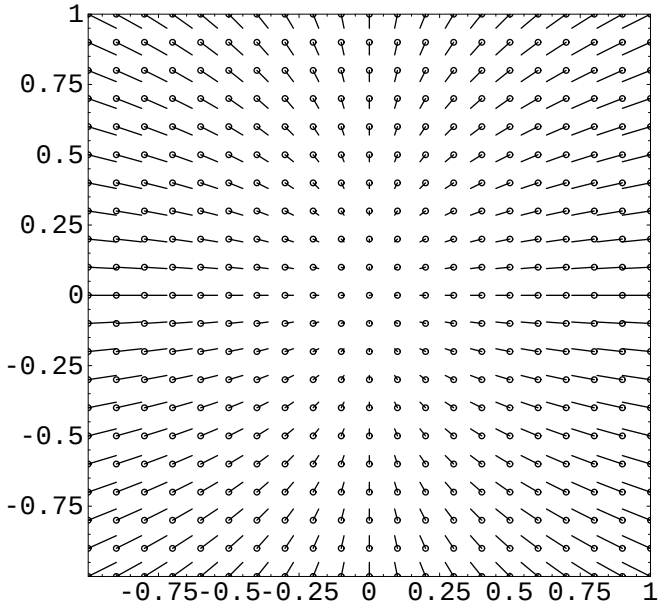
■ Sink/Source: $\lambda_1 < \lambda_2 < 0$ and $0 < \lambda_1 < \lambda_2$

Here the matrix is in JCF

```
(A = {{-2, 0}, {0, -1}}) // MatrixForm
MatrixExp[A t] // MatrixForm
ShowVF[A]
```

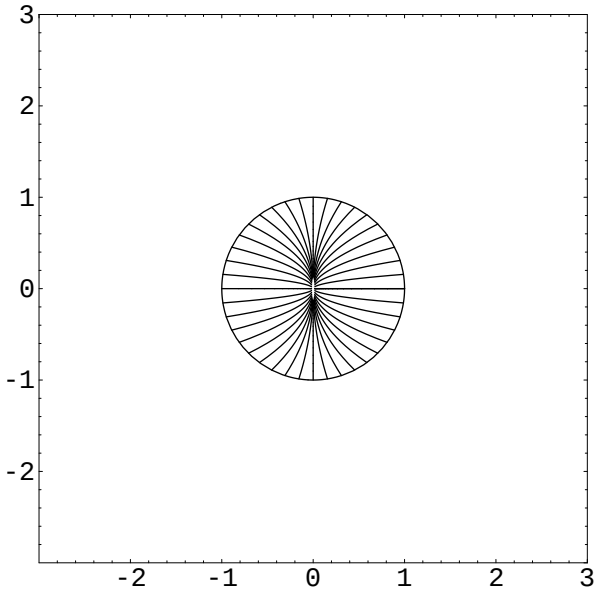
$$\begin{pmatrix} -2 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{pmatrix}$$



- Graphics -

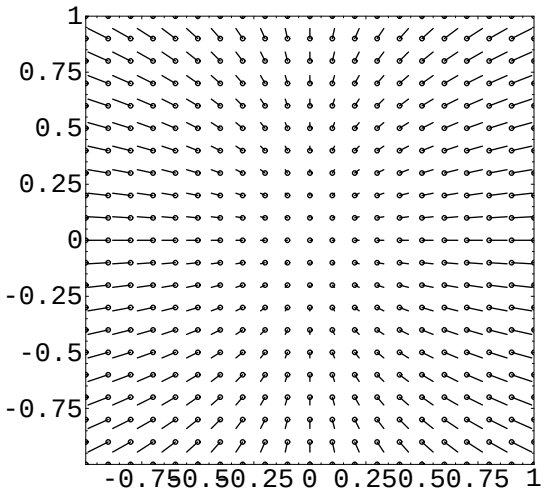
```
ShowFlow[A, 2]
```



- Graphics -

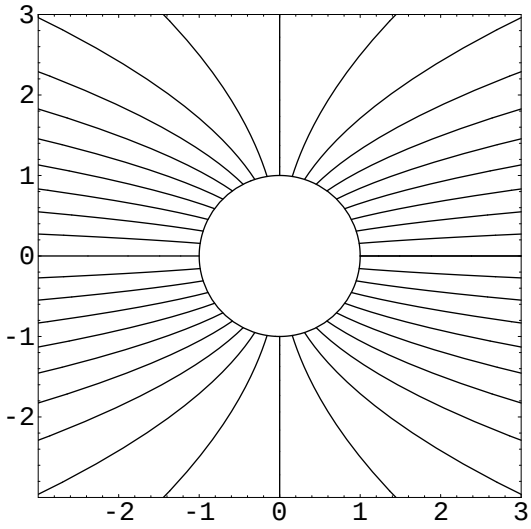
```
(-A) // MatrixForm
ShowVF[-A]
```

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$



- Graphics -

```
ShowFlow[-A, 2]
```



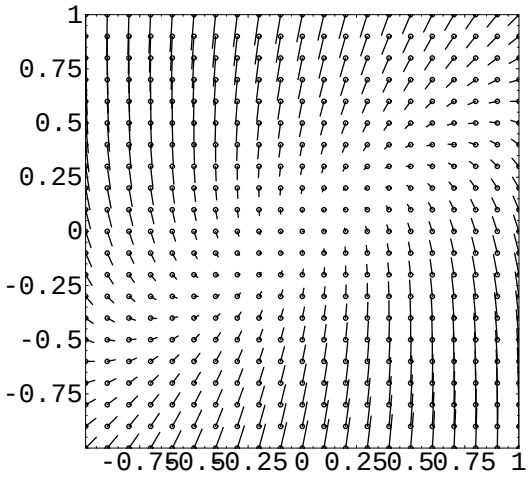

```
- Graphics -
```

Same flow, but relative to non−standard basis

```
(J = {{-2, 0}, {0, -1}});
P = {{1, 1}, {3, 1}};
A = P . J . Inverse[P];
A // MatrixForm
MatrixExp[A t] // MatrixForm
ShowVF[A]
```

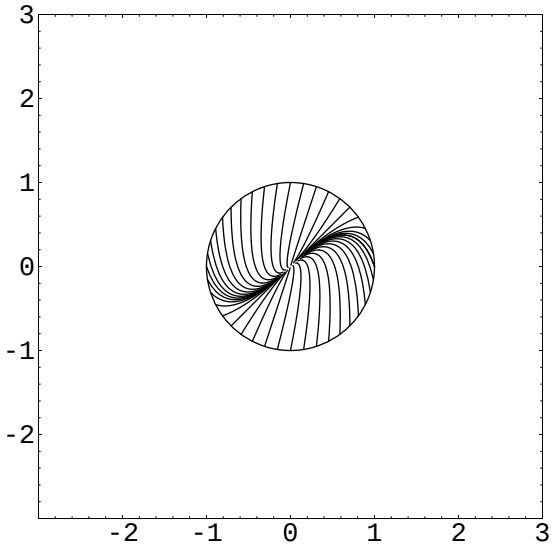
$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & -\frac{5}{2} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} e^{-2t} (-1 + 3 e^{t}) & -\frac{1}{2} e^{-2t} (-1 + e^{t}) \\ \frac{3}{2} e^{-2t} (-1 + e^{t}) & -\frac{1}{2} e^{-2t} (-3 + e^{t}) \end{pmatrix}$$



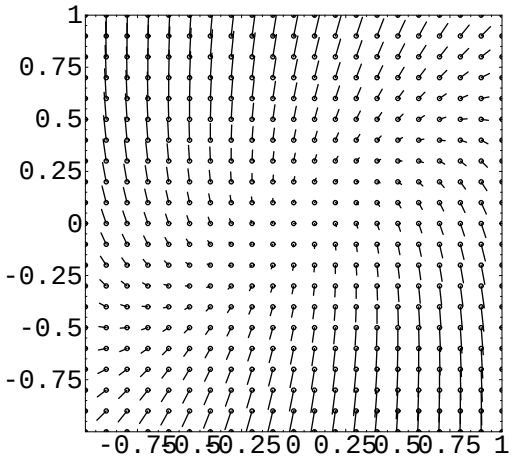

```
- Graphics -
```

ShowFlow[A, 2]



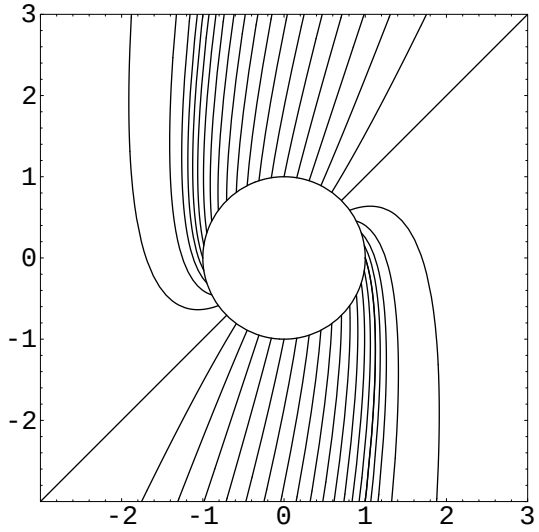
- Graphics -

ShowVF[-A]



- Graphics -

ShowFlow[-A, 2]



- Graphics -

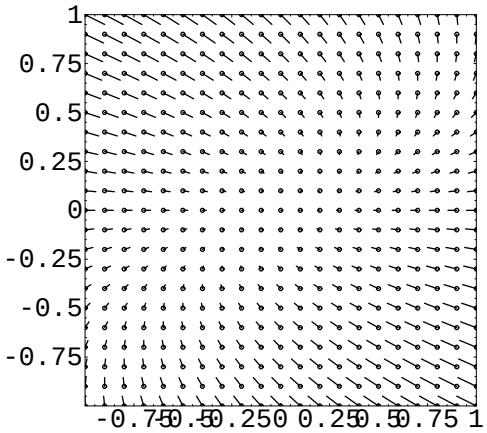
■ Singular Sink/Source: $\lambda_1 = \lambda_2 < 0$ and $0 < \lambda_1 = \lambda_2$

Here the matrix is in JCF

```
(A = {{-1, 1}, {0, -1}}) // MatrixForm
MatrixExp[A t] // MatrixForm
ShowVF[A]
```

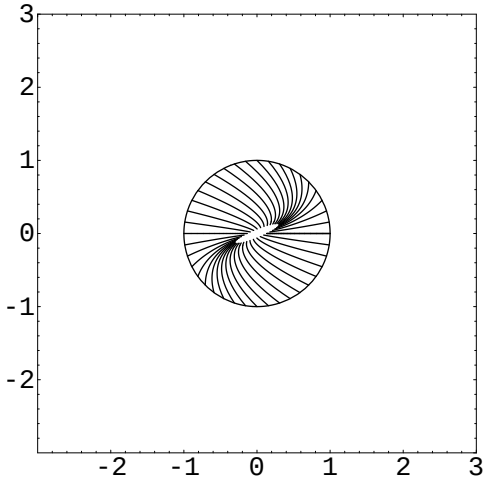
$$\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} e^{-t} & e^{-t}\, t \\ 0 & e^{-t} \end{pmatrix}$$



- Graphics -

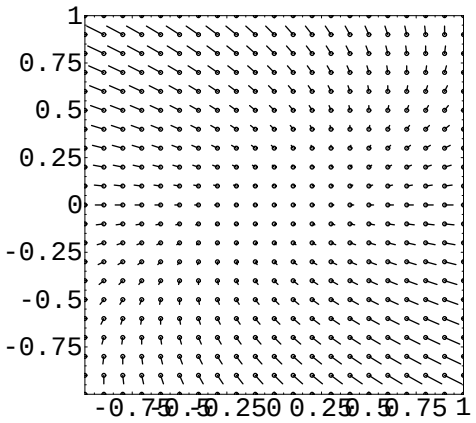
```
ShowFlow[A, 2]
```



- Graphics -

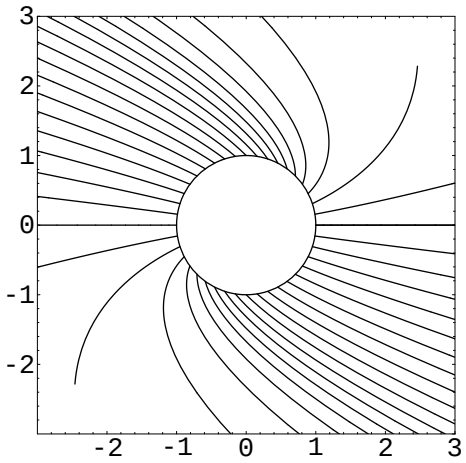
```
(-A) // MatrixForm
ShowVF[-A]
```

$$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$



- Graphics -

```
ShowFlow[-A, 2]
```



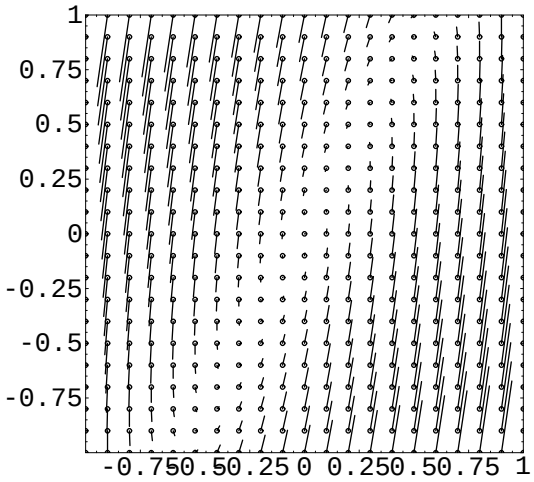
- Graphics -

Same flow, but relative to non−standard basis

```
(J = {{-1, 1}, {0, -1}});
P = {{1, 1}, {3, 1}};
A = P . J . Inverse[P];
A // MatrixForm
MatrixExp[A t] // MatrixForm
ShowVF[A]
```

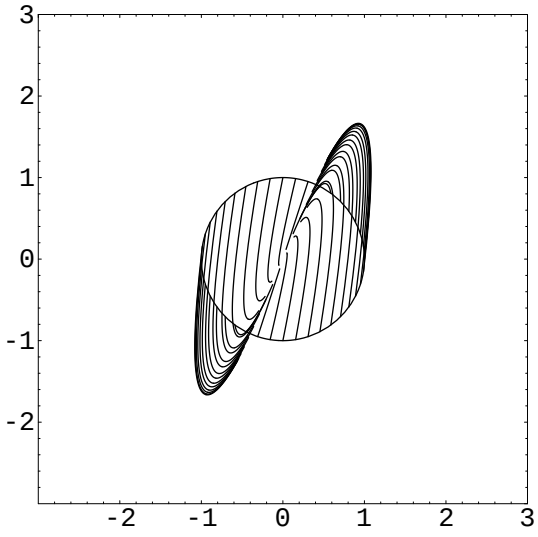
$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{9}{2} & -\frac{5}{2} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} e^{-t} (2+3t) & -\frac{1}{2} e^{-t} t \\ \frac{9 e^{-t} t}{2} & -\frac{1}{2} e^{-t} (-2+3t) \end{pmatrix}$$



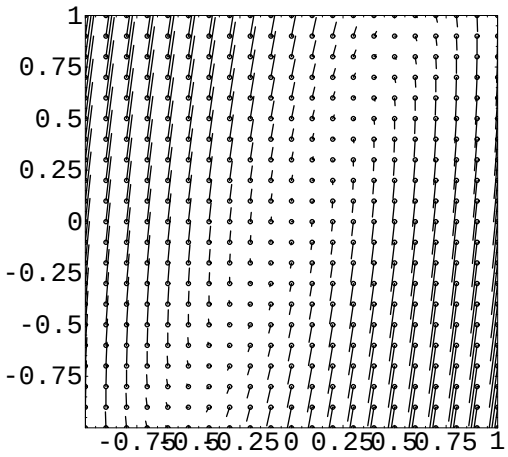
- Graphics -

```
ShowFlow[A, 2]
```



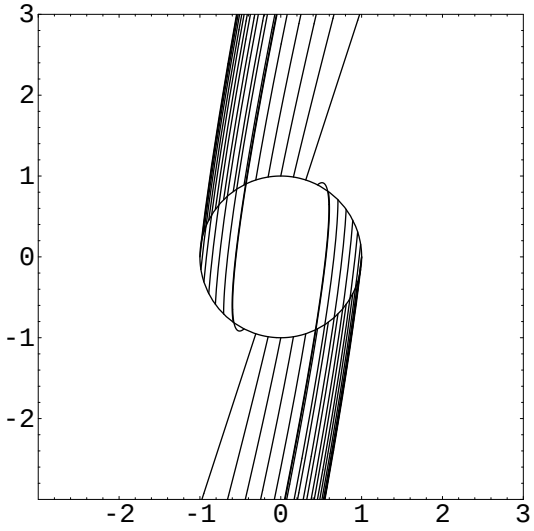
- Graphics -

ShowVF[-A]



- Graphics -

ShowFlow[-A, 2]



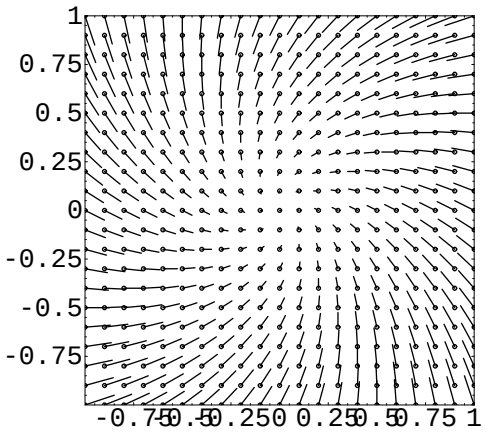
- Graphics -

■ Stable/Unstable focus: Re[λ] < 0, Im[λ] ≠ 0 Re[λ]>0, Im[λ]≠0

```
A = {{-2, -1}, {1, -2}};
A // MatrixForm
MatrixExp[A t] // MatrixForm
ShowVF[A]
```

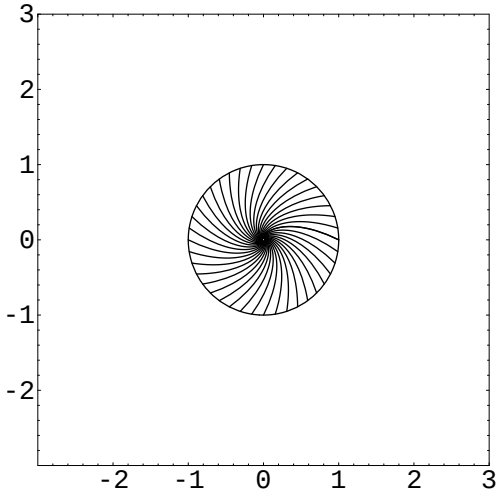
$$\begin{pmatrix} -2 & -1 \\ 1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} e^{-2t}\,\text{Cos}[t] & -e^{-2t}\,\text{Sin}[t] \\ e^{-2t}\,\text{Sin}[t] & e^{-2t}\,\text{Cos}[t] \end{pmatrix}$$



- Graphics -

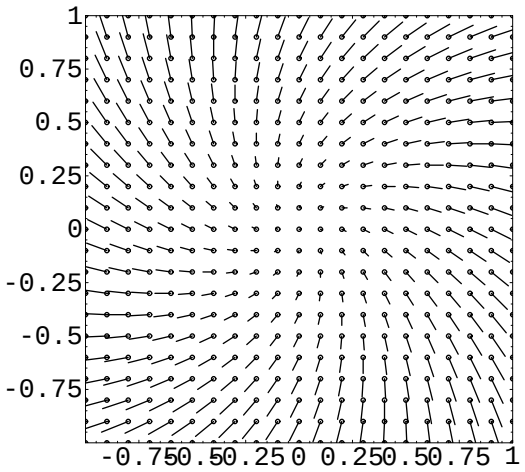
```
ShowFlow[A, 2]
```



- Graphics -

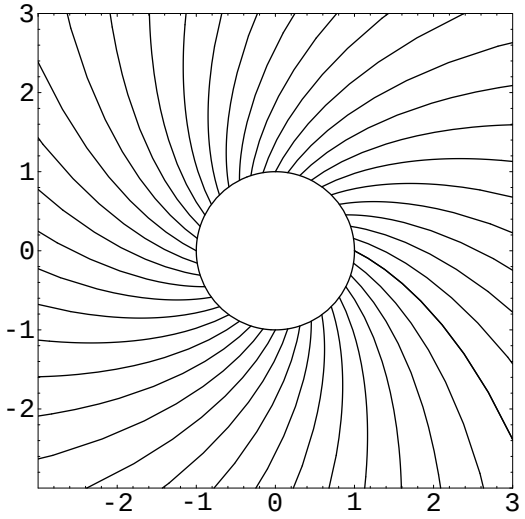
```
(-A) // MatrixForm
ShowVF[-A]
```

$$\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$$



- Graphics -

```
ShowFlow[-A, 2]
```



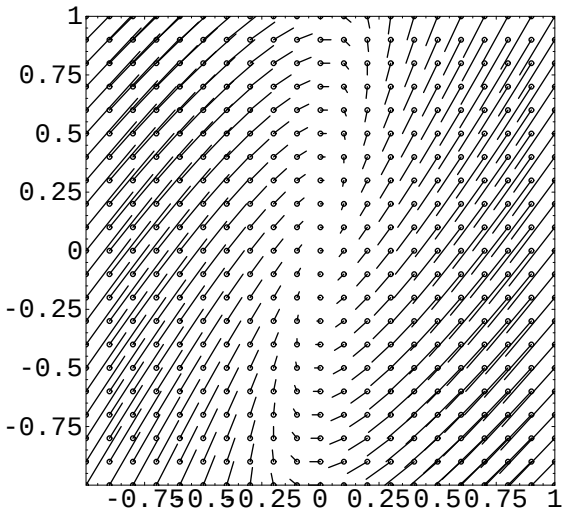
- Graphics -

Use a non−standard basis

```
J = {{-2, -1}, {1, -2}};
P = {{1, 1}, {3, 1}};
A = P . J . Inverse[P];
A // MatrixForm
MatrixExp[A t] // MatrixForm
ShowVF[A]
```

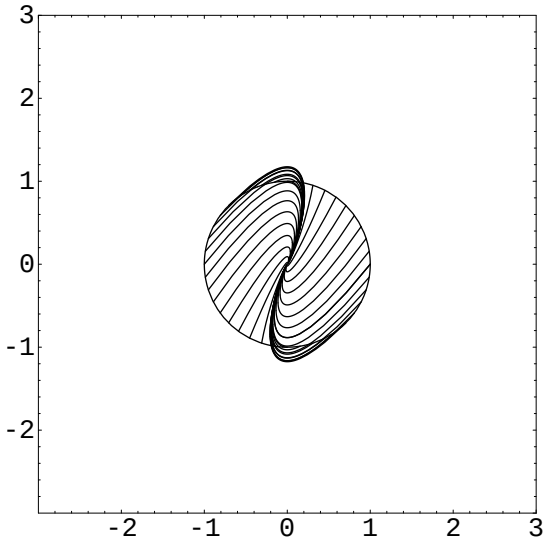
$$\begin{pmatrix} -4 & 1 \\ -5 & 0 \end{pmatrix}$$

$$\begin{pmatrix} e^{-2t}\,(\text{Cos}[t] - 2\,\text{Sin}[t]) & e^{-2t}\,\text{Sin}[t] \\ -5\,e^{-2t}\,\text{Sin}[t] & e^{-2t}\,(\text{Cos}[t] + 2\,\text{Sin}[t]) \end{pmatrix}$$



- Graphics -

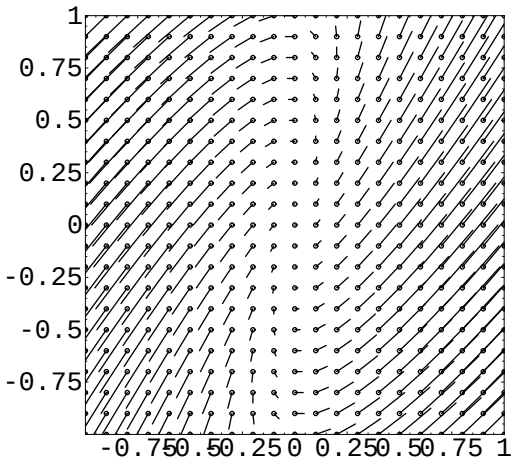
```
ShowFlow[A, 2]
```



- Graphics -

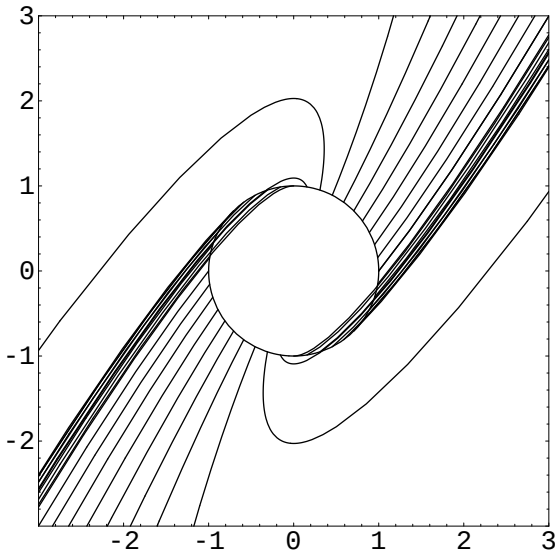
```
(-A) // MatrixForm
ShowVF[-A]
```

$$\begin{pmatrix} 4 & -1 \\ 5 & 0 \end{pmatrix}$$



- Graphics -

```
ShowFlow[-A, 2]
```



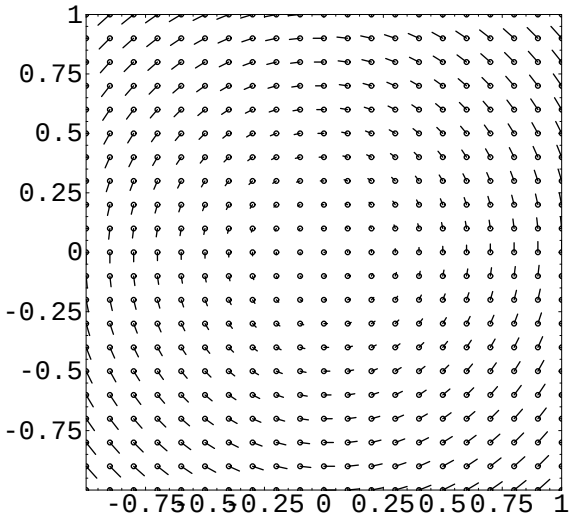
- Graphics -

■ Centre: Re[λ] = 0, Im[λ] ≠ 0 Im[λ]≠0

```
A = {{0, -1}, {1, 0}};
A // MatrixForm
MatrixExp[A t] // MatrixForm
ShowVF[A]
```

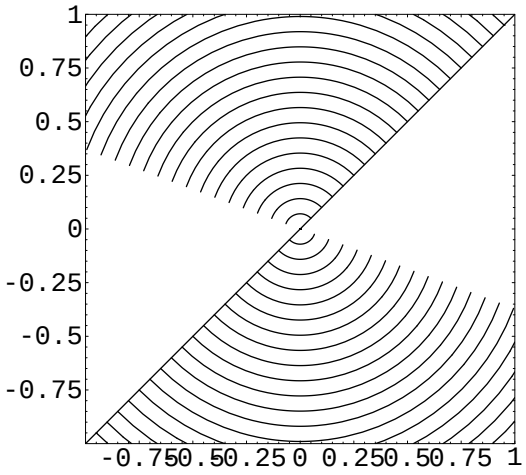
$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[t] & -\text{Sin}[t] \\ \text{Sin}[t] & \text{Cos}[t] \end{pmatrix}$$



- Graphics -

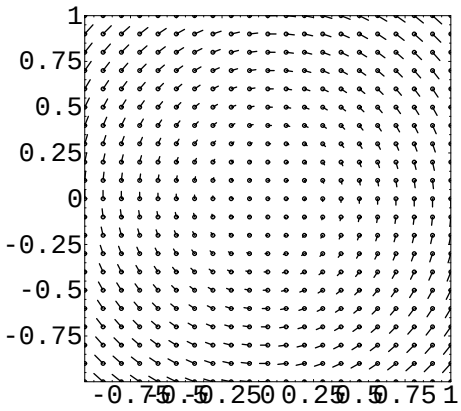
```
ShowFlowLine[A, 2, 40, -1, 1]
```



- Graphics -

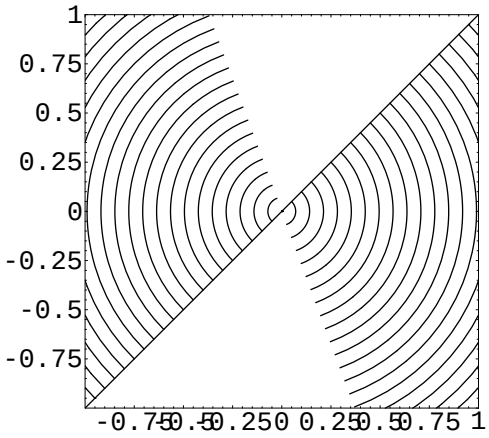
```
(-A) // MatrixForm
ShowVF[-A]
```

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$



- Graphics -

```
ShowFlowLine[-A, 2, 40, -1, 1]
```



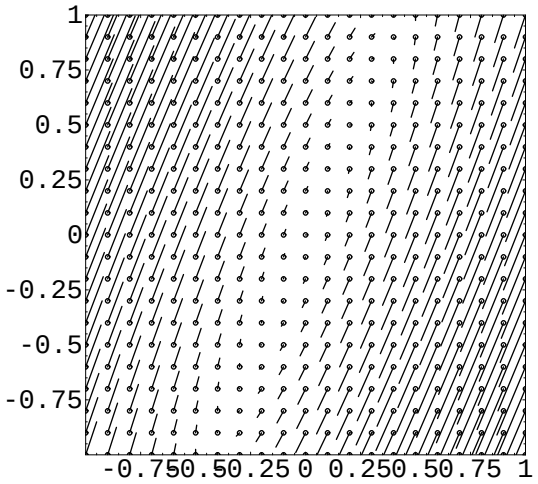
- Graphics -

Use a non−standard basis

```
J = {{0, -1}, {1, 0}};
P = {{1, 1}, {3, 1}};
A = P . J . Inverse[P];
A // MatrixForm
MatrixExp[A t] // MatrixForm
ShowVF[A]
```

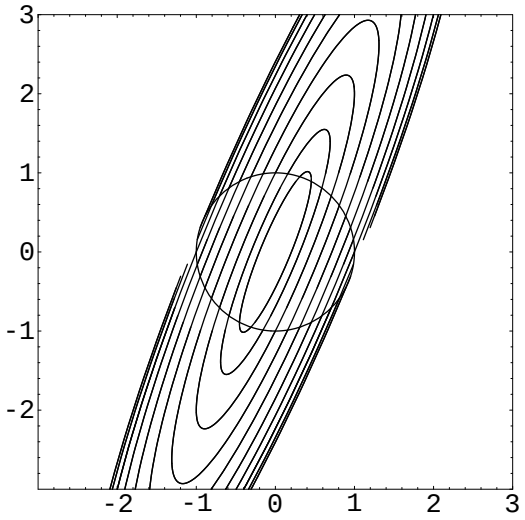
$$\begin{pmatrix} -2 & 1 \\ -5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} \text{Cos}[t] - 2\,\text{Sin}[t] & \text{Sin}[t] \\ -5\,\text{Sin}[t] & \text{Cos}[t] + 2\,\text{Sin}[t] \end{pmatrix}$$



- Graphics -

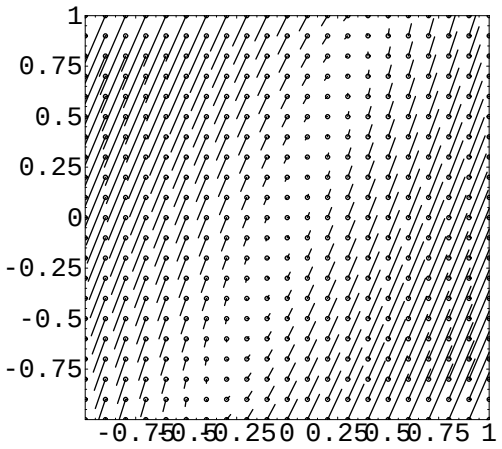
```
ShowFlow[A, 3, 40]
```



- Graphics -

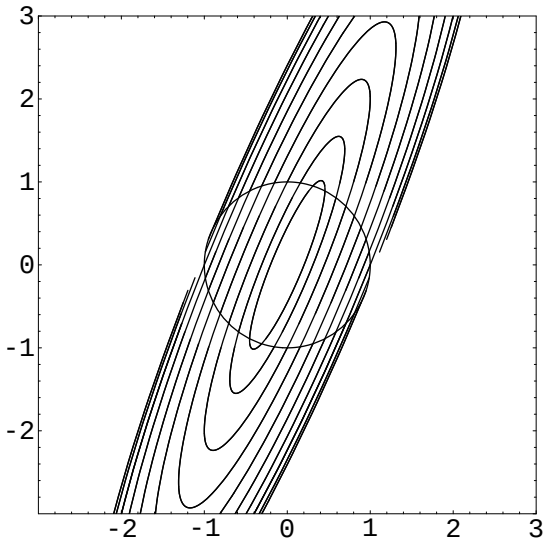
```
(-A) // MatrixForm
ShowVF[-A]
```

$$\begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$$



- Graphics -

```
ShowFlow[A, 3, 40]
```



- Graphics -