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On primes and terms of prime or 2^k index in the Lehmer sequences,
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Abstract

It is known that with a very small number of exceptions, for a term of a Lehmer sequence $\{U_n(\sqrt{R}, Q)\}$ to be prime its index must be prime. For example, $F_4 = U_4(1, -1) = 3$ is prime. Also, $U_n(1, 2)$ is prime for $n = 6, 8, 9, 10, 15, 25, 25, 65$, while $V_n(1, 2)$ is prime for $n = 9, 12$, and 20. This criterion extends to the companion Lehmer sequences $\{V_n(\sqrt{R}, Q)\}$, with the exception that primality may occur if the index is a power of two. Furthermore, given an arbitrary prime p or any positive integer k, there does not exist an explicit means for determining whether U_p, V_p, or V_{2^k} is prime. In 2000, V. Drobot provided conditions under which if p and $2p - 1$ are prime then F_p is composite. A short while later, L. Somer considered primes of the form $2p \pm 1$, as well as generalized Drobot's theorem to the Lucas sequences. Most recently, J. Jaroma extended Somer's findings to the companion Lucas sequences. In this paper, we shall generalize all of the aforementioned results from the Lucas sequences to the Lehmer sequences.