Douglas J. Shaw The pure numbers generated by the Collatz sequence, Fibonacci Quart. 44 (2006), no. 3, 194–201.

Abstract

The Collatz Conjecture has been vexing mathematicians for over seventy years. It is the only mathematics problem that is comprehensible to a fourth-grader, yet has caused Paul Erdős to say, "Mathematics is not yet ready for such a problem." This paper divides the integers into "pure" and "impure" numbers, where the "impure" numbers occur in the trajectories of smaller numbers. It develops an infinite set of theorems characterizing the "impure" numbers, and establishes bounds on their density in the integers.