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Continued fractions with partial quotients bounded in average,
Fibonacci Quart. **44** (2006), no. 4, 297–301.

Abstract

We ask, for which n does there exists a k, $1 \leq k < n$ and $(k, n) = 1$, so that k/n has a continued fraction whose partial quotients are bounded in average by a constant B? This question is intimately connected with several other well-known problems, and we provide a lower bound in the case of $B = 2$. The proof, which is completely elementary, involves a simple "shifting" argument, the Catalan numbers, and the solution to a linear recurrence.