

George M. Phillips

Index-doubling in sequences by Aitken acceleration,
Fibonacci Quart. **45** (2007), no. 4, 313–318.

Abstract

Let (x_n) denote a sequence which is linearly convergent to a limit x , and whose second differences are all nonzero. For a large class of such sequences (x_n) , the associated sequence (x_n^*) defined by

$$x_n^* = \frac{x_{n+1}x_{n-1} - x_n^2}{x_{n+1} - 2x_n + x_{n-1}}$$

converges to x faster than (x_n) . The derivation of this associated sequence is called Aitken acceleration. In a paper published in 1984 I showed that, with $x_n = F_{n+1}/F_n$,

$$\frac{x_{n+r}x_{n-r} - x_n^2}{x_{n+r} - 2x_n + x_{n-r}} = x_{2n}$$

for $1 \leq r < n$ so that, in particular, $x_n^* = x_{2n}$. Thus, we have an associated sequence that is a subsequence of the original sequence. A number of authors have followed up this result and in the present paper I summarize the progress made on this topic to date and present some new results.