John Greene
The Unboundedness of a Family of Difference Equations Over the Integers,
Fibonacci Quart. **46/47** (2008/2009), no. 2, 146–152.

Abstract

In this paper, we prove that positive integer solutions $\{a_n\}$ to

$$a_n = \begin{cases} \frac{c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}}{d}, & \text{if } d | c_1 a_{n-1} + \cdots + c_k a_{n-k}; \\ c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}, & \text{otherwise,} \end{cases}$$

where the c's are nonnegative integers, and $d = c_1 + c_2 + \cdots + c_k$, have the property that either $\{a_n\}$ is periodic with period at most k, or $\{a_n\}$ is unbounded.