Paul K. Stockmeyer
A Smooth Tight Upper Bound for the Fibonacci Representation Function $\boldsymbol{R(n)}$,
Fibonacci Quart. **46/47** (2008/2009), no. 2, 103–106.

Abstract

The function $R(n)$ that counts the number of representations of the integer n as the sum of distinct Fibonacci numbers has been studied for over 40 years, and many fascinating properties have been discovered. In this paper we prove that $R(n) \leq \sqrt{n+1}$ for all $n \geq 0$, with equality if and only if $n = F_m^2 - 1$ for some integer $m \geq 2$.