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**Abstract**

A general combinatorial approach is presented for proving identities of the form $mf_n = \sum_{i \in I_m} f_{n+i}$, where $m$ is a nonnegative integer constant, $n \geq |min(I_m)|$ is an integer, $I_m$ is a set of nonconsecutive integers, and $f_n$ is the Fibonacci number $F_{n+1}$. The approach involves counting phased square-domino tilings, as in the book *Proofs that Really Count* by Benjamin and Quinn. Furthermore, for each proof of an identity of the form $mf_n = \sum_{i \in I_m} f_{n+i}$, there is a corresponding isomorphic proof of the identity $m = \sum_{i \in I_m} \phi^i$, where $\phi$ is $(1 + \sqrt{5})/2$.